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Fooled by Info-Gap Decision Theory

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Preface

You can fool all the people some of the time and some of the people all of the time, but you cannot fool all the people all the time.

Abraham Lincoln (1809-1865)

In this rather long article I address a more specific issue regarding the scope of the "fooled by" phenomenon, namely

- How long, and how many times, will referees and area editors of risk analysis journals be fooled by the literature on info-gap decision theory?

Having followed the publications on info-gap decision theory (Ben-Haim 2001, 2006, 2010) for more than nine years now, I've long come to expect just about anything from the rhetoric in these publications. And yet, . . . for all that, it remains a puzzle to me that this rhetoric continues to pass muster in the peer-review process of risk analysis journals.

What I find so baffling is that peer-reviewed journals, for instance the journal *Risk Analysis,* continue to sanction a rhetoric that:

- Misrepresents as "novel", indeed as "radically different", a theory that in essence is no more and no less than a rehash of a number of well known and well established, mainstream concepts and ideas.
- Misrepresents the technical/mathematical facts pertaining not only to this theory, but also to other theories/methods, for instance worst-case analysis and Wald's maximin paradigm.
- Propounds egregious technical errors about basic mathematical concepts, as well as basic ideas in the broad area of decision-making, and the disciplines bearing on it, for instance, optimization theory, especially *robust optimization*.

The objective of this article is therefore to put before the readers of the risk analysis literature, especially readers of the journal *Risk Analysis,* the bare facts about info-gap decision theory. The idea is to make it clear to these readers that the rhetoric describing this theory and arguing for its application is grossly misleading.

To be precise, my objective is to clarify to these readers where precisely did referees and area editors of journals dealing with risk analysis go wrong, hence the title of this article.

And to illustrate what I have in mind, consider the short article: "*Fooled by local robustness*" (Sniedovich 2012) which was published recently in the journal *Risk Analysis.* In this article I explain, among other things, that info-gap decision theory is utterly unsuitable for the treatment of a **severe** uncertainty of the type that this theory claims to address. Because, as I show in this article, the severe uncertainty that info-gap decision theory claims to address calls for the use of models of **global** robustness, whereas info-gap's robustness model is by definition a model of **local** robustness.

And yet ... in the same issue of the journal, Hall et al. (2012) not only repeat the fallacy that info-gap robustness is not a **local** robustness. They compound this fallacy with the absurd explanation that info-gap decision theory is able to provide a **global** robustness because it has the ability to control the scope of its robustness analysis from local to global by varying the size of the uncertainty set (neighborhood) over which the robustness analysis is conducted. This preposterous claim should have been identified for what it is in the peer-review process.

But, it was not!

So, in this article I show that this claim exhibits a profound misapprehension of the concept **global robustness**. Specifically, I show that Hall et al. (2012) confuse two different types of analysis, namely:

- A **global robustness analysis** of an **uncertainty parameter** with respect to a **performance constraint.**
- A **parametric local robustness analysis** with respect to a **critical level of performance.**

The present article expands my analysis of the "fooled by" factor outlined in Sniedovich (2012a), by applying this analysis to the **entire theory.**

This expanded analysis of the "fooled by" factor exposes other errors, misconceptions and fallacies in the rhetoric of info-gap publications, which alas, continues to mislead referees and area editors. One such example is the claim that info-gap decision theory has the ability to **maximize the probability of success/survival** without using probabilistic information.

Another example is the claim that minimax/maximin models require the uncertainty to be **bounded.** Oddly enough, this claim is made in Ben-Haim (2012a), which was published in the same recent issue of *Risk Analysis* alongside Sniedovich (2012), which provides a formal rigorous proof that info-gap's robustness model is no more and no less than a simple instance of Wald maximin model. My point is that the absurd in Ben-Haim's (2012a) claim ought to have been identified in the peer-review process. Because, what this claim effectively entails, among many other things, is that a real-valued function cannot be optimized (in a global sense) over the real line. For instance, this claim effectively rules out the optimization (in a global sense) of the value of $\sin(x)$ over the real line.

Suffice it to say that if the claim that minimax/maximin models require the uncertainty to be **bounded** had any merit whatsoever, it would have been imperative to immediately radically modify the syllabus of first year college/university courses and to rewrite numerous textbooks in math, engineering, economics, physics, management, finance, and more.

To sum it all up then, the objective of this article is to demonstrate that, as attested by Volume 32, Issue 10 of *Risk Analysis,* referees and area editors continue to prove gullible to the misleading rhetoric in the literature on info-gap decision theory.

A formal, rigorous critique of info-gap decision theory can be found in Sniedovich (2007, 2008, 2009, 2010, 2012, 2012a, 2012b).

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1 Introduction

An examination of the peer-reviewed literature on risk analysis and decision-making under a non-probabilistic uncertainty reveals that the theory that exhibits the greatest confidence in its ability to provide a reliable tool for the management of a severe, non-probabilistic, unbounded uncertainty, is no doubt . . . *info-gap decision theory* (Ben-Haim 2001, 2006, 2010). Indeed, the literature on *info-gap decision theory* abounds with breathtaking assertions about the theory's ability to take on a severe/deep/radical, non-probabilistic, unbounded uncertainty. For instance, according to Wintle et al. (2010), the theory can even handle *Black Swans* and *Unknown Unknowns!*

Small wonder then that this theory is mentioned, albeit obliquely, in the article *"Confronting Deep Uncertainties in Risk Analysis"*, that was published recently in the journal *Risk Analysis:*

> Decision making without knowledge of, or agreement about, the basic assumptions needed to structure a decision problem by specifying a unique decision model, M, has been studied under headings such as deep uncertainty,[56] severe uncertainty,[3] model uncertainty, and wicked decision problems.[72]
>
> Cox (2012, p. 1610)

Observe that reference (3) is the book "*Information-gap decision theory: decisions under severe uncertainty*" (Ben-Haim 2001).

That said, it is nevertheless surprising that this theory is not included in Cox's (2012, Table I, p. 1611) short-list of ". . . 10 tools that can help us to better understand deep uncertainty and make decisions even when correct models are unknown . . . ". This is surprising, because no other theory, or method, proposing a strategy, approach etc., for the treatment of a severe/deep, non-probabilistic, unbounded uncertainty seems to have had such a presence in the two well established peer-reviewed risk analysis journals cited in Table 1 (below).

It is important to note that advocates of info-gap decision theory take great pains to emphasize that, unlike other theories, this theory allows the uncertainty to be **unbounded**:

> There is also a continuous and unbounded version of nesting intervals in an approach known as info-gap analysis that would be useful if one cannot develop finite bounds on some of the inputs.
>
> SAB (EPA) (2011, p. 37)

Indeed, according to the Father of info-gap decision theory,

> Most of the commonly encountered info-gap models are unbounded.
>
> Ben-Haim (2001, p. 208 ; 2006, p. 210)

> We will encounter many examples of info-gap models of uncertainty. In all cases an info-gap model is an unbounded family of nested sets of possible realizations.
>
> Ben-Haim (2010, p. 7)

And consider the following claim, which seems to attribute info-gap decision theory unique, indeed extraordinary, powers to deal with severe uncertainty:

Table 1: Summary of articles on info-gap decision theory in risk analysis(((??? journals ???)))

Title	Journal	year
Value-at-risk with info-gap uncertainty	JRF	2005
Managing credit risk with info-gap uncertainty	JRF	2007
Trading indicators with information-gap uncertainty	JRF	2008
Wald's maximin model: a treasure in disguise!	JRF	2008
An info-gap approach to managing portfolios of assets with uncertain returns	JRF	2009
Gearing investments with uncertainty	JRF	2010
A bird's view of info-gap decision theory	JRF	2010
Book review	RISA	2005
Managing the risk of uncertain threshold responses: comparison of robust, optimum, and precautionary approaches	RISA	2007
Evaluating critical uncertainty thresholds in a spatial model of forest pest invasion risk	RISA	2009
Robustness of risk maps and survey networks to knowledge gaps about a new invasive pest	RISA	2010
Reconciling uncertain costs and benefits in Bayes Nets for invasive species management	RISA	2010
Interpreting null results from measurements with uncertain correlations: an info-gap approach	RISA	2011
Confronting deep uncertainties in risk analysis	RISA	2012
Using expert judgments to explore robust alternatives for forest management under climate change	RISA	2012
Modeling Extreme Risks in Ecology	RISA	2012
Doing our best: optimization and the management of risk	RISA	2012
Why risk analysis is difficult, and some thoughts on how to proceed	RISA	2012
Robust climate policies under uncertainty: a comparison of info-gap and RDM methods	RISA	2012
Modeling Extreme Risks in Ecology	RISA	2012
Fooled by local robustness	RISA	2012
A new multi-criteria risk mapping approach based on a multi-attribute frontier concept	RISA	2013

RISA = *Risk Analysis,* JRF = *Journal of Risk Finance*

> A third, less common way to propagate uncertainty through mathematical expressions is embodied in an info-gap analysis [1]. The essence of this approach that distinguishes it from more traditional uncertainty analyses is that it doesn't require the analyst to circumscribe the uncertainty all in one fell swoop with finite characterizations having known bounds. Rather than committing to some horizon of uncertainty, the analyst expresses the problem as a *series of nested uncertainty analyses,* the bounds of which may grow without limit. In this respect, the info-gap approach is acknowledging that the uncertainty could always be worse. There are always additional questions one might ask, further doubt one might harbor. By expressing an infinity of such questions and doubts in a unified calculation, the approach goes beyond a classical Knightian view of decision under uncertainty.
>
> Ferson and Tucker (2008, p. 1)

And how about this?

> Making Responsible Decisions (When it Seems that You Can't)
> Engineering Design and Strategic Planning Under Severe Uncertainty
>
> What happens when the uncertainties facing a decision maker are so severe that the assumptions in conventional methods based on probabilistic decision analysis are untenable? Jim Hall and Yakov Ben-Haim describe how the challenges of really severe uncertainties in domains as diverse as climate change, protection against terrorism and financial markets are stimulating the development of quantified theories of robust decision making.
>
> Hall and Ben-Haim (2007, p. 1)

To be sure, an uncertainty that is non-probabilistic, likelihood-free, and on top of it unbounded, is indeed **severe.**

However ...

The important point to note in this regard is that it is one thing to proclaim a method/theory singularly suitable for the treatment of a severe/deep/radical uncertainty by claiming that this method/theory allows its uncertainty it to be non-probabilistic, likelihood-free, and on top of it unbounded. But, it is quite another to demonstrate that the method/theory in question can indeed "deliver the goods". In other words, the mere fact that a method/theory allows its uncertainty to be non-probabilistic, likelihood-free, and unbounded does not imply that this method/theory is indeed able to handle this uncertainty. To make a case for such a claim, it is imperative to show that the method/theory concerned actually has the technical capabilities that are required to deal properly with such an uncertainty.

In the case of info-gap decision theory, there is a yawning gap between the rhetoric in the literature on the theory, which claims to provide a reliable tool for this type of uncertainty, and the hard technical facts about this theory's mode of operation.

Thus, although it is no doubt true that the uncertainty stipulated by info-gap decision theory is severe, the fact of the matter is that the theory lacks the technical wherewithal required to meet the challenges posed by such a severe uncertainty. For instance, both methodologically and practically, info-gap's robustness analysis does nothing to deal with the fact that the uncertainty is **unbounded.** Indeed, this fact is completely **ignored** in its robustness analysis. By the same token, although the theory is claimed to be **non-probabilistic** and **likelihood-free,** it nevertheless prescribes no more than a **local** robustness analysis in the **neighborhood** of the **point estimate** of the uncertainty parameter. It thus assigns a far greater weight/importance to the immediate neighborhood of the point estimate than to any other neighborhood in the unbounded uncertainty space, and this without the slightest justification, or explanation, or argument.

Add to this the fact that articles on info-gap decision theory, published in the risk analysis literature, are also replete with utterly unfounded claims about the theory's role and place in the state of the art, and the question naturally arising is this:

- How is it that articles advocating the use of this theory continue to pass muster in the peer-review process of journals devoted to risk analysis, including the journal *Risk Analysis?*

The answer to this question is simple: this is due to a failure in the peer-review process to recognize and properly judge assertions that:

- Portray info-gap decision theory as "radically different" from well-established, mainstream theories.

- Misrepresent the state of the art in decision-making under non-probabilistic uncertainty.
- Misrepresent the hard facts about the technical characteristics of info-gap decision theory's core mathematical models, as well as those of mainstream paradigms for decision-making under non-probabilistic uncertainty.

The overall conclusion to be drawn from this discussion is then that short of a close familiarity with info-gap decision theory, one would not be able to see through the misleading rhetoric in the literature on info-gap decision theory. Meaning that a fleeting familiarity with this theory is not sufficient to judge correctly how and why this rhetoric misrepresents the hard facts about this theory's mode of operation, its capabilities, its scope, and its role and place in the state of the art.

I should point out that most of the technical aspects of the issues raised in this article have already been brought to light and discussed in detail in an array of publications (e.g. Sniedovich 2007, 2008, 2009, 2010, 2012, 2012a, 2012b). The purpose of this article is to pull them together so as to give a full explanation of how, despite the hard facts about the theory having been available in the peer-reviewed literature since 2007, referees of peer-reviewed journals, such as *Risk Analysis,* continue to be taken in by its rhetoric.

For an immediate sense of what is in store in this article, let us first take a quick look at the latest news from the *Risk Analysis/Info-gap* literature.

2 The latest rhetoric

An example of the uncritical acceptance of rhetoric about info-gap decision theory can be found in a recent issue of the journal *Risk Analysis*. Here the referees' acquiescence in a rhetoric designed to corroborate the claim that info-gap decision theory is "radically different" from mainstream theories for decision under non-probabilistic uncertainty, prevented them from detecting a gross technical error that is implies by this rhetoric.

To see what I have in mind, consider the following extract from the lengthy discussion in Ben-Haim (2012a, pp. 1644-1645) on the difference between min-max robustness and info-gap robustness. It asserts, without proof or argument, that while maximin models require the uncertainty to be **bounded,** info-gap decision theory allows its uncertainty to be **unbounded!** (emphasis added):

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst. **The min-max concept responds to severe uncertainty that nonetheless can be bounded.** The info-gap concept responds to severe uncertainty that is **unbounded** or whose **bound is unknown.**
>
> Ben-Haim (2012a, p. 1644)

The implication is then that the following simple minimax model ought to be deemed invalid:

$$z^* := \min_{-\infty<x<\infty} \ \max_{-\infty<u<\infty} \{x^2 + 2xu - u^2\} \tag{1}$$

observing that in this model both the decision variable (x) and the uncertainty parameter (u) are **unbounded.**

But:

The fact of the matter is of course that this is a perfectly kosher, simple minimax model. Its optimal solution is $(x^*, u^*) = (0, 0)$, hence $z^* = 0$. Namely, contrary to Ben-Haim's (2012a) claim, this minimax model is perfectly capable of handling an unbounded uncertainty. The question is then this:

- Given the preeminent position of Wald's maximin paradigm in decision theory, how is it that a peer-reviewed journal, such as *Risk Analysis,* accepted for publication an article that so grossly misrepresents the basic properties of this stalwart of decision theory?

Even more perplexing is the fact that this claim appears in an article that was published alongside, namely in the same issue as, Sniedovich (2012a), where it is proven formally and rigorously that info-gap's robustness model and info-gap's robust-satisficing decision model are simple maximin models. Because, the contradiction that this claim gives rise to is this:

- While maximin models require their uncertainty to be **bounded,** info-gap's robustness model, which is a simple maximin model, allows its uncertainty parameter to be **unbounded!**

Suffice it to say that contradictions such as this, that are inherent in the rhetoric on info-gap decision theory, abound in the literature. It is essential therefore to sort out the hard facts from the misleading rhetoric about it to enable a correct assessment of its capabilities and limitations, and its role and place in the state of the art. To do this it is essential to give an accurate picture of info-gap decision theory's two core mathematical models, namely its robustness model and its robust-satisficing decision model.

Let us then examine these two core models.

3 Info-gap decision theory: first encounter

Info-gap decision theory (Ben-Haim 2001, 2006, 2010) is a non-probabilistic decision theory that is concerned with decision problems involving three main ingredients:

- A *decision variable* $q \in Q$.
- An *uncertainty parameter* $u \in \mathscr{U}$.
- A *performance requirement* of the following form:

$$r_c \leq r(q, u) \tag{2}$$

  where r_c is a numeric scalar representing a critical *performance level* and $r(q, u)$ is a numeric scalar representing the performance level of decision q given u. Formally, regard r as a real-valued function on $Q \times \mathscr{U}$.

A crucial assumption here is that the true value of the uncertainty parameter u is unknown, indeed it is assumed to be subject to a **severe** uncertainty. All we know about the true value of u is that it is an element of some given set $\mathscr{U}$. We shall refer to this set as the *uncertainty space*.

The theory's objective is therefore to identify decisions that are **robust** against the severe uncertainty in the true value of u, namely against the variation in the value of u over $\mathscr{U}$. The robustness of decision $q \in Q$, as defined by this theory, is supposed to be a measure of the decision's ability to satisfy the performance requirement $r_c \leq r(q,u)$ in the face of the severe uncertainty in the true value of u, namely as u varies over $\mathscr{U}$.

Two extreme situations might arise in this pursuit to satisfy the performance constraint $r_c \leq r(q,u)$ for values of u in $\mathscr{U}$:

- Decision q is **super-robust:** $r_c \leq r(q,u), \forall u \in \mathscr{U}$.
- Decision q is **super-fragile:** $r_c > r(q,u), \forall u \in \mathscr{U}$.

As a general rule, though, decisions are typically neither super-robust nor are they super-fragile. Typically, decision q satisfies the performance constraint for some, but not all, u in $\mathscr{U}$. Therefore, the key methodological question arising here is this:

- How should we **define** the robustness of decision q with respect to the performance constraint $r_c \leq r(q,u)$ against variations in the value of u over $\mathscr{U}$?

That said, it is important to point out straightaway that info-gap decision theory **does not concern itself with this key question,** which needless to say, is inexplicable, considering its claims to be concerned with a truly severe uncertainty. Instead, the question that info-gap decision theory does addresses is this:

- How should we define the **local** robustness of decision q with respect to the performance constraint $r_c \leq r(q,u)$ against **perturbations in a given nominal value of** u?

The nominal value of u, call it $\tilde{u} \in \mathscr{U}$, represents a **point estimate** of the true value of u. So the **local** robustness question addressed by info-gap decision theory can be phrased as follows:

- How robust is decision q with respect to the performance constraint $r_c \leq r(q,u)$ against the uncertainty in the true value of u **in the neighborhood of the point estimate** $\tilde{u}$?

The approach that info-gap decision theory puts forth to answer this question, and to determine the best (optimal) decision, is dubbed in the info-gap literature "robust-satisficing" (Ben-Haim 2006, 2010, 2012, 2012a). This approach deploys two simple mathematical models whose structures are transparently clear. One model defines the info-gap robustness of decisions, the other determines the best (most robust) decision.

Therefore, the only meaningful way to clarify where info-gap's robust-satisficing approach to severe uncertainty stands vis-a-vis mainstream approaches to severe uncertainty, is to compare the two **mathematical models** deployed by info-gap decision theory to this end, with mathematical models representing mainstream approaches to severe uncertainty, and let the ... **mathematics do the talking.**

The two core mathematical models that info-gap's robust-satisficing approach to severe uncertainty is based on are these (Ben-Haim 2001, 2006, 2010):

Info-gap's robustness model	Info-gap's robust-satisficing decision model
$\hat{\alpha}(q,\tilde{u}) := \max_{\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	$\hat{\alpha}(\tilde{u}) := \max_{q\in Q} \hat{\alpha}(q,\tilde{u})$ $= \max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

where $U(\alpha,\tilde{u}) \subseteq \mathscr{U}$ denotes a **neighborhood** of size α around the point estimate $\tilde{u}$. In words:

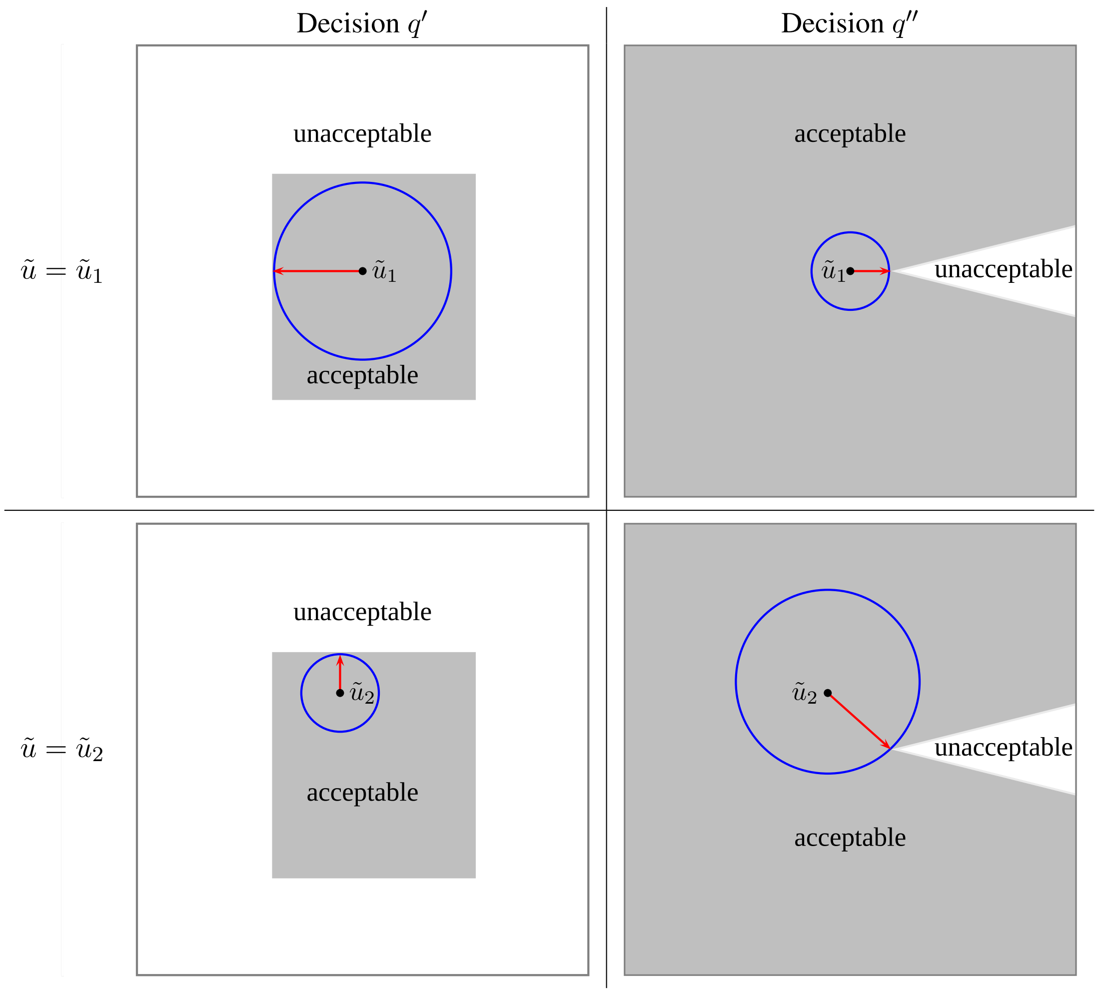

Figure 1: The info-gap robustness of two decisions q' and q'' at two point estimates $\tilde{u}_1$ and $\tilde{u}_2$.

- The info-gap robustness of decision q at $\tilde{u}$, denoted $\hat{\alpha}(q, \tilde{u})$, is equal to the size ($\alpha$) of the largest neighborhood $U(\alpha, \tilde{u})$ around $\tilde{u}$ such that the performance constraint $r_c \leq r(q, u)$ is satisfied for all values of u in this neighborhood.
- The optimal decision is a decision whose info-gap robustness is the largest.
- Differently put, the info-gap robustness of decision q at $\tilde{u}$ is the **distance (α) of the point estimate $\tilde{u}$ to infeasibility.** Namely, it is the size α of the **smallest perturbation** in $\tilde{u}$ such that, if increased, it will violate the performance constraint $r_c \leq r(q, u)$. Formally, $\hat{\alpha}(q, \tilde{u})$ is equal to the smallest value of α such that $r(q, u) > r_c$ for some $u \in U(\alpha', \tilde{u})$, $\forall \alpha' > \alpha$.

The definition of info-gap robustness is illustrated in Figure 1, where the large rectangles represent the uncertainty space $\mathscr{U}$, the shaded areas represent the sets of acceptable values of u, namely values of u that satisfy the performance constraint $r_c \leq r(q, u)$, the circles represent the largest neighborhoods contained in the shaded areas, and the radii of these circles represent the info-gap robustness of the decisions under consideration. This figure makes vivid that the info-gap robustness of a decision may vary as the value of the point estimate $\tilde{u}$ varies.

Another fact made vivid by this figure is that, according to the precepts of info-gap decision theory, changes in the value of the point estimate $\tilde{u}$ can cause a **preference reversal**. Note, for instance, that decision q' is more info-gap-robust than decision q'' at $\tilde{u} = \tilde{u}_1$, whereas decision q'' is more info-gap robust than decision q' at $\tilde{u} = \tilde{u}_2$.

These, needless to say, are immediate consequences of info-gap's robustness model being a model of **local** robustness, hence a reminder that info-gap robustness is a measure of the **local** robustness of decisions in the **neighborhood** of the point estimate $\tilde{u}$.

Severe uncertainty

Info-gap decision theory (Ben-Haim 2001, 2006) stipulates that the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ are subsets of the uncertainty space $\mathscr{U}$ and that they satisfy two basic conditions:

$$U(0, \tilde{u}) = \{\tilde{u}\} \text{ (contraction)} \tag{3}$$

$$\alpha' < \alpha'' \longrightarrow U(\alpha', \tilde{u}) \subseteq U(\alpha'', \tilde{u}) \text{ (nesting).} \tag{4}$$

With no loss of generality we assume that the uncertainty space $\mathscr{U}$ is the smallest set such that $U(\alpha, \tilde{u}) \subseteq \mathscr{U}, \forall \alpha \geq 0$, the implication being that these neighborhoods expand with α, and that at the limit, namely as $\alpha \to \infty$, the neighborhood $U(\alpha, \tilde{u})$ spans the entire uncertainty space $\mathscr{U}$.

It is important to note, though, that as such, these properties of the neighborhoods do not imply that the uncertainty postulated by info-gap decision theory is **severe**. Rather, the properties that jointly render this uncertainty **severe** are the following:

- The uncertainty space $\mathscr{U}$ is vast, indeed typically it is **unbounded.**
- The point estimate $\tilde{u}$ is a **poor** indication of the true value of u and can therefore be **substantially wrong.** It is a kind of **guess,** sometimes just a **wild guess.**
- The quantification of the uncertainty is **non-probabilistic, likelihood-free, belief-free,** etc.

The latter implies, among other things, that there are no grounds to assume that the true value of u is more/less likely to be in the immediate vicinity of the point estimate $\tilde{u}$ than in the immediate vicinity of any other value of u in $\mathscr{U}$.

It is also important to take note that much as the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ around the point estimate $\tilde{u}$ expand with α and α is unbounded above, info-gap's robustness analysis of decision q is typically confined to a **bounded** neighborhood $U(\alpha', \tilde{u})$, where α' is **fixed.** To be precise, as a rule, info-gap's robustness analysis of decision q is not conducted over the entire uncertainty space $\mathscr{U}$: It is confined to a bounded neighborhood $U(\alpha', \tilde{u})$ where $\alpha' = \hat{\alpha}(q, \tilde{u}) + \varepsilon$ and $\varepsilon > 0$ is arbitrarily small. Obviously, this is dictated by info-gap's **definition** of the robustness of decision $q \in Q$ at $\tilde{u}$, which as you recall, is as follows:

$$\hat{\alpha}(q, \tilde{u}) := \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\}. \tag{5}$$

As can be ascertained **by inspection** from this definition, insofar as decision q is concerned, the admissible values of α are bounded above by $\hat{\alpha}(q, \tilde{u})$. Hence, by definition, $\hat{\alpha}(q, \tilde{u})$ is equal to the largest admissible value of α associated with decision q at $\tilde{u}$.

The implication is therefore clear. The info-gap robustness of decision q at $\tilde{u}$, namely $\hat{\alpha}(q,\tilde{u})$, is determined in total disregard of the performance levels $r(q,u)$ associated with values of u that are outside the neighborhood $U(\alpha',\tilde{u})$, where $\alpha' = \hat{\alpha}(q,\tilde{u}) + \varepsilon$ and $\varepsilon > 0$ can be arbitrarily small. Hence,

THEOREM 3.1 *The info-gap robustness of decision q, namely $\hat{\alpha}(q,\tilde{u})$, is* INVARIANT *with the uncertainty space $\mathscr{U}$, so long as $U(\alpha',\tilde{u}) \subseteq \mathscr{U}$, where $\alpha' = \hat{\alpha}(q,\tilde{u}) + \varepsilon$ and $\varepsilon > 0$ can be arbitrarily small. In other words, the performance levels $r(q,u)$ associated with values of u outside the neighborhood $U(\alpha',\tilde{u})$ have no impact whatsoever on the info-gap robustness of decision q at $\tilde{u}$.*

Proof. This follows immediately from the definition of $\hat{\alpha}(q,\tilde{u})$ and the fact that the neighborhoods $U(\alpha,\tilde{u})$, $\alpha \geq 0$ are nested. That is, consider a given decision $q \in Q$ and let $\alpha' = \hat{\alpha}(q,\tilde{u}) + \varepsilon$ for some arbitrarily small (positive) ε. Since $\alpha' > \hat{\alpha}(q,\tilde{u})$, it follows that for every $\alpha \geq \alpha'$ there is a $u \in U(\alpha,\tilde{u})$ such that $r_c > r(q,u)$. This implies that the value of $\hat{\alpha}(q,\tilde{u})$ is independent of values of $r(q,u)$ pertaining to values of u outside the neighborhood $U(\alpha',\tilde{u})$. In fact, if you contract $\mathscr{U}$ and change it to $\mathscr{U}' = U(\alpha',\tilde{u})$ and then recalculate the value of $\hat{\alpha}(q,\tilde{u})$, you'll get the same value. □

For this reason, Sniedovich (2010, 2012a, 2012c) refers to

$$NML(q,\tilde{u},\varepsilon) := \mathscr{U} \setminus U(\alpha',\tilde{u}) \ , \ \alpha' = \hat{\alpha}(q,\tilde{u}) + \varepsilon \tag{6}$$

as the *No Man's Land* of decision q.

Observe then that info-gap's robustness analysis of decision q at $\tilde{u}$ **takes no account whatsoever** of the values of u in the set $NML(q,\tilde{u},\varepsilon)$. It is as though these values do not exist and therefore have no effect on the value of $\hat{\alpha}(q,\tilde{u})$.

It is important to appreciate the ramifications of this fact. By the same token that the uncertainty space $\mathscr{U}$ of an info-gap robustness model, which, as trumpeted in the info-gap literature, is typically **unbounded** above, in view of what we saw above, the *No Man's Land* $NML(q,\tilde{u},\varepsilon)$ of decision q, is equally typically **unbounded.** And the implication of this is that info-gap's robustness analysis **typically ignores most of the possible/plausible values of the uncertainty parameter** u. It is as though these values do not exist and therefore have no effect on the value of $\hat{\alpha}(q,\tilde{u})$.

Typically then, in the local robustness analysis of decision q at $\tilde{u}$ the value of α is bounded above by $\alpha' = \hat{\alpha}(q,\tilde{u}) + \varepsilon$. This means that in effect $\mathscr{U}$ is **typically bounded,** namely $\mathscr{U} = U(\alpha',\tilde{u})$, to further imply that this set is typically infinitesimally small, relative to the unbounded uncertainty space under consideration. This is illustrated in Figure 3.

In the next section we look at three mainstream measures of robustness that are relevant to this discussion.

4 The state of the art

To be able to assess correctly where info-gap's robustness analysis fits in the state of the art, one must examine it vis-a-vis the **mainstream** non-probabilistic models of robustness. Such an examination is important because the literature on info-gap decision theory would have us believe that info-gap decision theory is radically different from all current theories for decisions under uncertainty. The three models that are relevant to such an examination are these:

- Wald's maximin model (circa 1940)
- Radius of stability model (circa 1960)
- Domain/volume/size model (circa 1963)

As we shall see, all three models are based on a **worst-case** approach to uncertainty: For a decision to be admissible, it must satisfy the performance constraints for **all** the possible/plausible values of the uncertainty parameter under consideration. In other words, an admissible decision is a decision that proves resilient to the worst-case scenario pertaining to the assumed set of values of the uncertainty parameter. This implies that both the radius of stability model and the domain/volume/size model are maximin models.

We examine these models in a reverse chronological order.

4.1 Domain/volume/size model

This eminently logical/intuitive measure of robustness gauges the "size" or "volume" of the set of acceptable values of the parameter of interest. In *decision theory,* it has its roots in Starr's (1963, 1966) *Domain* criterion. To relate it to info-gap robustness, assume that we have a set of decisions $q \in Q$, an uncertainty parameter $u \in \mathscr{U}$, and a number of constraints on the (q, u) pairs. Let $A(q)$ denote the set of *acceptable* values of u associated with decision q, that is let

$$A(q) := \{u \in \mathscr{U} : \text{ The pair } (q, u) \text{ satisfies the constraints under consideration}\}\ ,\ q \in Q. \tag{7}$$

Note that in the case of info-gap decision theory, these sets are stated as follows:

$$A(q) = \{u \in \mathscr{U} : r_c \leq r(q, u)\}\ ,\ q \in Q. \tag{8}$$

According to the "domain/size/volume" criterion, the robustness of decision q against the uncertainty in the true value of u is equal to the "size" of set $A(q)$: the larger this set, the more robust decision q. Thus, let $SIZE(V)$ denote the "size" of set $V \subseteq \mathscr{U}$ according to some suitable measure of "size". For instance, if the sets $A(q), q \in Q$ consist of finitely many elements, we can let $SIZE(A(q)) = |A(q)|$ where $|A(q)|$ denotes the cardinality of set $A(q)$. So, let the "size" robustness of decision q be defined as follows:

$$\text{size-rob}(q) := SIZE(A(q))\ ,\ q \in Q \tag{9}$$

$$= \max_{V \subseteq \mathscr{U}} \{SIZE(V) : u \in A(q), \forall u \in V\}. \tag{10}$$

In words, the size-robustness of decision q, denoted size-rob(q), is equal to the size of set $A(q)$, which in turn is equal to the size of the largest subset of $\mathscr{U}$, all whose elements satisfy the performance constraints under consideration. Note that according to this measure of robustness, the larger SIZE(A(q)), the more robust decision q.

In the case of the constraint deployed by info-gap decision theory, namely $r_c \leq r(q, u)$, we thus have

$$\text{size-rob}(q) = \max_{V \subseteq \mathscr{U}} \{SIZE(V) : u \in A(q), \forall u \in V\} \tag{11}$$

$$= \max_{V \subseteq \mathscr{U}} \{SIZE(V) : r_c \leq r(q, u), \forall u \in V\}. \tag{12}$$

It is important to appreciate the fundamental difference between the info-gap robustness of decision q and the size-robustness of decision q. Thus, whereas the former is determined by the size of the **largest neighborhood around** $\tilde{u}$ that is contained in $A(q)$, the latter is determined by the size of the **largest subset of** $\mathscr{U}$ that is contained in $A(q)$, which of course, is the seize of $A(q)$ itself. The implication of this fact is clear: info-gap robustness is a measure of **local** robustness, whereas the size-robustness is a measure of **global** robustness.

Consequently, the fact that the info-gap robustness of decision q' is, say, larger than the info-gap robustness of decision q'', does not imply that $Size(A(q')) > Size(A(q''))$, and vice versa. This is illustrated in Figure 1 where the sets of acceptable values of u are represented by the shaded areas. Hence, $SIZE(A(q))$ is equal to the size of the shaded area representing acceptable values of u pertaining to decision q.

Observe that the worst-case orientation of this measure of robustness is manifested in the iconic constraint

$$u \in A(q), \forall u \in V \tag{13}$$

which signifies that, for the given subset $V \subseteq \mathscr{U}$ under consideration to be admissible, **all** the values of $u \in V$ must be acceptable with respect to decision q.

Alas, for all its intuitive appeal, the application of this measure of global robustness is quite limited, because in practice it often proves exceedingly difficult to efficiently determine/compute the size of the sets $A(q), q \in Q$. There are, of course, many exceptions, e.g. Starr 1963, 1966; Schneller and Sphicas 1983; Rosenblat 1987; Eiselt and Langley 1990; Eiselt and Laporte 1992; Eiselt et al. 1998; Vetschera, 2009.

4.2 Radius of stability model

There are many situations where robustness is sought against **small perturbations** in a **nominal value** of a parameter. In such cases, the robustness of a decision is defined as the size of the **smallest perturbation** in the nominal value such that, the tiniest increase in this perturbation will destabilize the decision in question. Hence, the larger this critical perturbation, the more robust the decision. Differently put, the radius of stability of decision q is the radius (size) of the **largest neighborhood** around the nominal value of the parameter, all whose elements perform satisfactorily with respect to the conditions imposed on the decision.

This measure of local robustness/stability is known universally as **radius of stability** (circa 1960) and it is used extensively in many fields (e.g. Wilf 1960, Milne and Reynolds 1962, Hindrichsen and Pritchard 1986, Zlobec 1987, Paice and Wirth 1998, Anderson and Bernfeld 2001, Cooper et al. 2004, Raab and Feroz 2007).

Formally, the radius of stability of decision $q \in Q$ at $\tilde{u}$ would be define as follows:

$$\hat{\rho}(q, \tilde{u}) := \max_{\alpha \geq 0} \{\alpha : con(q, u), \forall u \in U(\alpha, \tilde{u})\} \ , \ q \in Q \tag{14}$$

where $con(q, u)$ denotes a list of conditions (constraints) imposed on (q, u) pairs, and $U(\alpha, \tilde{u})$ denotes a neighborhood of size (radius) α around the nominal point $\tilde{u}$.

In fact, this concept of local robustness/stability is so rudimentary that one might term it **intuitive.** And to be sure, over the years, it had been appealed to (independently) by many scholars, working in various areas of endeavor, and, depending on the area in question, this concept was given various appellations so as to reflect the application considered. For instance, in Dontchev et al. (2003) it is called *radius of regularity*.

Indeed, even those who are not mathematically savvy, should be able to work out its meaning even in cases where the mathematical framework in which it is invoked is unfamiliar to them. To illustrate, consider this:

> It is convenient to use the term "radius of stability of a formula" for the radius of the largest circle with center at the origin in the s-plane inside which the formula remains stable.
>
> Milne and Reynolds (1962, p. 67)

And even this:

4.9. Radius of Stability

> Consider the following problem: Suppose that the model (P, ϑ) is stable at some $\vartheta = \vartheta^*$. Determine the largest open sphere $S(\vartheta^*, r)$, centered at ϑ^*, with the property that the model is stable at every point of $S(\vartheta^*, r)$. The radius r of the sphere is termed the radius of stability (see [33]). Knowledge of this radius is important, because it, tells us how far one can uniformly strain the (engineering, economic) system before it begins to break down. Presently, there does not seem to exist an efficient method for calculating the radius of stability, even for bi-linear models. ■
>
> Zlobec (1987, p. 326)

And how about this?

> Robustness analysis has played a prominent role in the theory of linear systems. In particular the state-state approach via stability radii has received considerable attention, see [HP2], [HP3], and references therein. In this approach a perturbation structure is defined for a realization of the system, and the robustness of the system is identified with the norm of the smallest destabilizing perturbation. In recent years there has been a great deal of work done on extending these results to more general perturbation classes, see, for example, the survey paper [PD], and for recent results on stability radii with respect to real perturbations, see [QBR$^+$]. In some cases this approach has even lead to algorithms for designing controllers for maximum robustness [HP4]. To date, the problem of extending these results to nonlinear systems has received little attention, although local stability analysis for nonlinear systems based on the linearization around a fixed point is well known, see, e.g., [V]. One approach in this direction is presented in [CK], where time varying perturbations are considered.
>
> Paice and Wirth (1998, p. 289)

And this?

> For an efficient NG, the infinity-norm measure, or the radius of stability (herein termed stability index), defines the largest "cell" in which all simultaneous detrimental perturbations to the input and output components will not cause a change in the efficiency status from technically efficient to inefficient. As such, the larger the stability index, the more robustly efficient the NG is said to be. Those efficient NGs with small stability indices will thus become technically inefficient, with smaller detrimental perturbations than those efficient NGs with larger stability indices.
>
> Raab and Feroz (2007, p. 400)
>
> Note: NG = National Government

What is of immediate interest to this discussion is the fact that a comparison of (5) and (14) reveals, **by inspection**, that info-gap's robustness model is a simple radius of stability model. Hence, for the record:

THEOREM 4.2 *Info-gap's robustness model is a simple radius of stability model. Specifically, info-gap's robustness model is the instance of (14) characterized by the list of constraints, namely $con(q,u)$, consisting of the single constraint $r_c \leq r(q,u)$.*

Proof. Let $con(q,u)$ be the singleton consisting of the single constraint $r_c \leq r(q,u)$. In this case (14) yields (5). **QED**

It is important to keep in mind that the concept *radius of stability* designates a property that is inherently **local.** This is so because the perturbations/deviations are measured from a nominal value $\tilde{u}$ where the latter constitutes a benchmark or point of origin. Therefore, a variation in the value of $\tilde{u}$ stands to change the values of $\hat{\rho}(q,\tilde{u}), q \in Q$. And the implication is that the ranking of decisions will vary according to their radii of stability.

The worst-case orientation of this measure of robustness is manifested in the iconic

$$con(q,u), \forall u \in U(\alpha,\tilde{u}) \tag{15}$$

requirement.

That is, under the worst-case scenario approach, we require decision q to satisfy the constraints in $con(x,u)$ for **all** u in $U(\alpha,\tilde{u})$. This worst-case orientation is **local** in the sense that, as in the framework of info-gap decision theory, the domain of the analysis are neighborhood $U(\alpha,\tilde{u}), \alpha \geq 0$ around $\tilde{u}$, rather than the entire uncertainty space $\mathscr{U}$.

Therefore, unsurprisingly, the *No Man's Land* effect that characterizes info-gap's robustness analysis is present here as well.

4.3 Wald's maximin paradigm

The preeminent approach to non-probabilistic uncertainty offered by *decision theory* is no doubt Wald's **maximin** paradigm (Wald 1939, 1945, 1959, Luce and Raiffa 1957, Rawls 1971, Resnik 1987, French 1988, Sniedovich 2008). Recall that this paradigm ranks *alternatives* according to their **worst outcomes,** hence an optimal alternative is one whose worst outcome is at least as good as the worst outcomes of all other alternatives.

For the purposes of this discussion, it is instructive to consider a generic maximin model which seeks robustness with respect to both **payoffs/rewards** and performance **constraints**. So consider this generic maximin model

$$z^* := \max_{x \in X} \min_{s \in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \tag{16}$$

where

$$X = \text{set of } \textit{alternatives} \text{ available to the decision maker.} \tag{17}$$
$$S(x) = \text{set of } \textit{states} \text{ associated with } \textit{alternative } x \in X. \tag{18}$$
$$con(x,s) = \text{ list of } \textit{constraints} \text{ imposed on the } (x,s) \text{ pairs.} \tag{19}$$
$$f(x,s) = \text{ numeric scalar representing the } \textit{payoff/reward} \text{ generated by alternative } x \text{ and state } s. \tag{20}$$

Regard f as a real-valued function on $X \times \mathbb{S}$, where $\mathbb{S} := \cup_{x\in X} S(x)$. We shall refer to f as the *payoff/reward function* and to $\mathbb{S}$ as the *state space.*

In the framework of this model, the **outcome** generated by an (alternative, state) pair (x,s) is determined by two things, namely by (a) the payoff/reward $f(x,s)$ and (b) the constraints listed in $con(x,s)$. A payoff $f(x,s)$ is admissible iff the pair (x,s) satisfies the constraints imposed on this pair. Formally then, the outcome generated by (x,s) can be defined as follows:

$$outcome(x,s) := \begin{cases} f(x,s) & , \ (x,s) \text{ satisfies the constraints in } con(x,s) \\ -\infty & , \ (x,s) \text{ does not satisfy the constraints in } con(x,s) \end{cases} , x \in X, s \in S(x) \tag{21}$$

The penalty $-\infty$ signifies that the (alternative,state) pair (x,s) is inadmissible because it violates the constraints in $con(x,s)$.

Note that the generic maximin model (16) a priori discards all the alternatives that are inadmissible in the **worst-case sense,** namely alternatives $x \in X$ such that the constraints in $con(x,s)$ are violated for at least one $s \in S(x)$. Also note that for any worst-case admissible alternative $x \in X$, we have $outcome(x,s) = f(x,s), \forall s \in S(x)$. Therefore, the implicit assumption in (16) is that there is at least one worst-case admissible alternative, namely there exists at least one alternative $x \in X$ that satisfies the constraints imposed on x for all $s \in S(x)$. Therefore, in terms of "outcomes", the maximin model (16) can be written as follows:

$$z^\circ := \max_{x\in X} \min_{s\in S(x)} outcome(x,s). \tag{22}$$

The choice between these two equivalent maximin models, namely (16) and (22), is a matter of convenience. For the purposes of this discussion it is more convenient to use (16) because it explicitlyy displays the two iconic operations that characterize the maximin paradigm:

- The iconic inner $\min_{s\in S(x)}$ operation that represents **worst-case** robustness sought with respect to the **payoff/reward** $f(x,s)$.
- The iconic $\forall s \in S(x)$ clause that represents **worst-case** robustness sought with respect to the **performance constraints** listed in $con(x,s)$.

Readers who are at home with the modelling aspects of Wald's maximin paradigm, will no doubt see immediately that the two core models of info-gap decision theory are in fact simple maximin models. Namely, they will see at a glimpse that both models are simple instances of the generic maximin model defined in

(16). Still, for the benefit of all readers, let us verify this observation.

Observe then that all we have to do to prove formally and rigorously that the above two core info-gap models are indeed maximin models, is to identify the two instances of the generic maximin model (16) that correspond to these core info-gap models. So consider the two simple instances of (16) specified in Figure 2.

Generic maximin object	Instance I	Instance II
x	α	(q,α)
s	u	u
X	$[0,\infty)$	$Q\times[0,\infty)$
$S(x)$	$U(\alpha,\tilde{u})$	$U(\alpha,\tilde{u})$
$f(x,s)$	α	α
$con(x,s)$	$r_c\le r(q,u)$	$r_c\le r(q,u)$

Note: in *Instance I* the objects q and $\tilde{u}$ are fixed and given, and in *Instance II* the object $\tilde{u}$ is fixed and given.

Generic maximin model: $\max\limits_{x\in X}\ \min\limits_{s\in S(x)}\ \{f(x,s):con(x,s),\forall s\in S(x)\}$

Instance I: $\max\limits_{q\in Q}\ \{\alpha:r_c\le r(q,u),\forall u\in U(\alpha,\tilde{u})\}$

Instance II: $\max\limits_{q\in Q,\alpha\ge 0}\ \{\alpha:r_c\le r(q,u),\forall u\in U(\alpha,\tilde{u})\}$

Figure 2: A generic maximin model and two of its instances

THEOREM 4.3 *Info-gap's robustness model and info-gap's robust-satisficing decision model are simple maximin models. Specifically, both are simple instances of the generic maximin model given in (16).*

Proof. Substituting the specification of *Instance I* in the maximin model (16) yields the following simple maximin model:

$$z^* := \max_{x\in X}\ \min_{s\in S(x)}\ \{f(x,s):con(x,s),\forall s\in S(x)\} \tag{23}$$

$$= \max_{\alpha\ge 0}\ \min_{u\in U(\alpha,\tilde{u})}\ \{\alpha:r_c\le r(q,u),\forall u\in U(\alpha,\tilde{u})\} \tag{24}$$

$$= \max_{\alpha\ge 0}\ \{\alpha:r_c\le r(q,u),\forall u\in U(\alpha,\tilde{u})\}. \tag{25}$$

This is none other than info-gap's robustness model. And repeating the exercise with *Instance II*, yields the following simple maximin model:

$$z^* := \max_{x\in X}\ \min_{s\in S(x)}\ \{f(x,s):con(x,s),\forall s\in S(x)\} \tag{26}$$

$$= \max_{q\in Q,\alpha\ge 0}\ \min_{u\in U(\alpha,\tilde{u})}\ \{\alpha:r_c\le r(q,u),\forall u\in U(\alpha,\tilde{u})\} \tag{27}$$

$$= \max_{q\in Q,\alpha\ge 0}\ \{\alpha:r_c\le r(q,u),\forall u\in U(\alpha,\tilde{u})\} \tag{28}$$

which is none other than info-gap's robust-satisficing decision model. **QED**

It should come as no surprise that both the radius of stability model and the domain/volume/size robustness model are also maximin models (see Sniedovich 2011).

5 The big picture

To be able to appreciate why the rhetoric in the literature on info-gap decision theory is grossly misleading, it is important to have some idea of the extravagant assertions that are made in this literature about the capabilities of this theory and its place and role in the state of the art.

Indeed, the narrative in this literature would have us believe that this theory is radically different from mainstream non-probabilistic theories for decision under uncertainty furthermore, that the difference is real and deep (emphasis added):

> Info-gap decision theory is **radically different** from **all** current theories of decision under uncertainty. The difference originates in the modeling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with **severe lack of information** and **highly unstructured** uncertainty.
>
> Ben-Haim (2001, 2006, p. xii)

> In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are **real and deep.** Despite the power of classical decision theories, in many areas such as engineering, economics, management, medicine and public policy, a need has arisen for a different format for decisions based on **severely uncertain evidence.**
>
> Ben-Haim (2001, 2006, p. 11)

This narrative not only pits info-gap decision theory against probability theory, it claims to offer no less than a 'replacement' for probability theory:

> Probability and info-gap modeling each emerged as a struggle between rival intellectual schools. Some philosophers of science tended to evaluate the info-gap approach in terms of how it would serve physical science in place of probability. This is like asking how probability would have served scholastic demonstrative reasoning in the place of Aristotelian logic; the answer: not at all. But then, probability arose from challenges different from those faced the scholastics, just as the info-gap decision theory which we will develop in this book aims to meet new challenges.
>
> Ben-Haim (2001 and 2006, p. 12)

> The emergence of info-gap decision theory as a viable alternative to probabilistic methods helps to reconcile Knight's dichotomy between risk and uncertainty. But more than that, while info-gap models of severe lack of information serve to quantify Knight's 'unmeasurable uncertainty', they also provide new insight into risk, gambling, and the entire pantheon of classical probabilistic explanations. We realize the full potential of the new theory when we see that it provides new ways of thinking about old problems.
>
> Ben-Haim (2001 p. 304; 2006, p. 342)

> Info-gap decision theory clearly presents a 'replacement theory' with which we can more fully understand the relation between classical theories of uncertainty and uncertain phenomena themselves.
>
> Ben-Haim (2001 p. 305; 2006, p. 343)

And if this were not enough, the theory is claimed to be able to maximize the probability of success and minimize the likelihood of failure, although, in the same breath, it is proclaimed to be non-probabilistic and likelihood free (emphasis added):

> For example, the best management option may be one that ensures that a species does not exceed a given risk of extinction under the highest possible level of unfavorable uncertainty. The decision may not minimize the extinction risk when uncertainty is ignored, but it is the option **least likely to fail** because of uncertainty in model structure or parameter estimates.
>
> Nicholson and Possingham (2007, p. 252)

> One possible approach would be the application of tools from **non-probabilistic** decision theories, such as **info-gap decision theory** (Ben-Haim, 2006). Whereas classical decision theory approaches generally optimize the expected value of the decision variable, the **info-gap approach** instead **minimizes the probability of falling below a certain threshold** (i.e., it maximizes robustness to failure).
>
> Chisholm (2010, p. 1981)

We are advised that info-gap decision theory is capable of responding to an uncertainty so severe that it is outside the purview of Wald's maximin paradigm (emphasis added):

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst. **The min-max concept responds to severe uncertainty that nonetheless can be bounded.** The info-gap concept responds to severe uncertainty that is **unbounded** or whose **bound is unknown.**
>
> Ben-Haim (2012a, p. 1644)

Indeed there are claims in the literature on info-gap decision theory asserting that this theory **generalizes** the maximin strategy (emphasis added):

> Info-gap **generalizes** the maximin strategy by identifying worst-case outcomes at increasing levels (horizons) of uncertainty. This permits the construction of 'robustness curves' that describe the decay in guaranteed minimum performance (or worst-case outcome) as uncertainty increases.
>
> Wintle et al. (2011, p. 357)

And there are claims that analysts can change the scope of info-gap's robustness analysis from local to global (emphasis added):

> For small α, searching set $U(\alpha, \tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. **The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.**
>
> Hall et al. (2012, p. 1662)

These clearly are breathtaking claims.

The question therefore is:

- **What/where are the "proofs" substantiating the validity of these bold claims?**

The impression one gets is that, right from the outset, referees of articles on info-gap decision theory seem not to have insisted on solid "proofs" for these assertions, so that at this stage these unfounded claims have already taken root and are being repeated uncritically in the literature on risk analysis.

To the best of my knowledge, the first formal, rigorous examination of such claims appeared in the peer-reviewed literature in 2007, in the article: *"The art and science of modelling decision-making under severe uncertainty"* (Sniedovich 2007) and the first formal, rigorous examination of such claims appeared in the peer-reviewed risk analysis literature in 2010, in the article: *"A bird's view of info-gap decision theory"* (Sniedovich 2010). Yet despite the findings in Sniedovich (2007, 2010) and in other peer-reviewed articles on this subject (e.g. Sniedovich 2008, 2009, 2012, 2012a, 2012b), unfounded claims about info-gap decision theory continue to be bandied about in the peer-reviewed literature on risk analysis.

So in what follows, I once again explain why the hard facts about info-gap decision theory make a non-sense of the claims quoted above. To this end I focus on the following issues:

- **The worst-case analysis connection**
  An explanation why, contrary to the rhetoric in the literature on info-gap decision theory, info-gap's robustness analysis is a (local) worst-case analysis *par excellence*.
- **The maximin connection**
  An explanation why, contrary to the rhetoric in the literature on info-gap decision theory, info-gap's robustness model and info-gap's robust-satisficing decision model are simple maximin models.
- **The radius of stability connection**
  A reminder that info-gap robustness is a re-invention of the concept *radius of stability* (circa 1960), a well established and widely used measure of local robustness/stability.
- **Local vs global robustness**
  An analysis of erroneous claims in the info-gap literature denying that info-gap robustness is a measure of local robustness.
- **The robust optimization connection**
  An explanation of the apparent deliberate attempt in the literature on info-gap decision theory to dis-sociate info-gap's robust-satisficing approach to severe uncertainty from the long-established, thriving field of *robust optimization* (circa 1970).
- **The deterministic equivalents connection**
  An examination of the rhetoric in the literature on info-gap decision theory on the theory's claimed abil-ity to maximize the probability of success without using probabilistic information and its re-invention of the concept ‘’ deterministic equivalents” (circa 1960).

As detailed discussions on these issues appeared already in a number of articles, the discussions here are brief and serve primarily as pointers to these articles.

6 The worst-case analysis connection

The info-gap literature, e.g. Ben-Haim (2001, 2005, 2006, 2007, 2010, 2012a), is adamant that info-gap's robust-satisficing analysis is not a worst-case analysis.

> Since the horizon of uncertainty is unbounded, there is no worst case and the info-gap analysis cannot and does not purport to ameliorate a worst case.
>
> Ben-Haim (2005, p. 392)

> The uncertainty sets become more inclusive as the horizon of uncertainty h grows. There is no greatest horizon of uncertainty so there is no worst case in an info-gap model.
>
> Ben-Haim (2005, p. 399)

> The info-gap robustness function evaluates the greatest tolerable Knightian uncertainty. However, the info-gap robustness is not a minimax analysis (we do not ameliorate a worst case) since, in an info-gap model of uncertainty, there is no worst case: any given horizon of uncertainty is less severe than all greater horizons of uncertainty.
>
> Ben-Haim (2005, pp. 400-401)

> Optimization of the robustness in eq. (3.172) is emphatically not a worst-case analysis. In classical worst-case min-max analysis the decision maker minimizes the impact of the maximally damaging case. But an info-gap model of uncertainty is an unbounded family of nested sets: $U(\alpha,\tilde{u})$, for all $\alpha \geq 0$. Consequently, there usually is no worst case: any adverse occurrence is less damaging than some other more extreme event occurring at a larger value of α.
>
> Ben-Haim (2006, p. 101)

> Info-gap theory is not a worst-case analysis. While there may be a worst case, one cannot know what it is and one should not base one's policy upon guesses of what it might be. Info-gap theory is related to robust-control and min-max methods, but nonetheless different from them. The strategy advocated here is not the amelioration of purportedly worst cases.
>
> Ben-Haim (2010, p. 9)

The technical rationale put forward in Ben-Haim (2001, 2005, 2006, 2010) to justify these and similar claims made elsewhere, is that for a worst-case analysis to be applicable, worst outcomes must exist, and for this to be the case the domain of the analysis must be **bounded**. But since the info-gap uncertainty is unbounded, info-gap's robustness analysis cannot possibly be a worst-case analysis.

This egregiously erroneous rhetoric about worst-case analysis and info-gap's robustness analysis exhibits a profound misunderstanding of both analyses.

The point to note here is that although info-gap's uncertainty space $\mathscr{U}$ may well be allowed to be unbounded, the robustness analysis of decision q is not conducted over $\mathscr{U}$ itself, but rather over the neighborhoods $U(\alpha,\tilde{u}), 0 \leq \alpha \leq \hat{\alpha}(q,\tilde{u})$, **one neighborhood at a time**. Therefore, since each of these neighborhoods is bounded, the fact that $\mathscr{U}$ is unbounded does not imply that info-gap's robustness analysis is not a worst-case analysis. Rather, the implication is that info-gap's robustness analysis is a **local** worst-case analysis.

More fundamentally, it is important to appreciate that info-gap's robustness analysis of decision q is driven the **constraint**

$$r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u}) \tag{29}$$

and that this constraint is a **worst-case constraint**: It requires decision q to satisfy the constraint $r_c \leq r(q,u)$ for **all** values of u in the neighborhood $U(\alpha,\tilde{u})$, hence also for the **worst** u in this neighborhood (whatever it is).

Note that a worst-case analysis of this constraint is a priori guaranteed to generate at **least one worst case**, hence the existence of a worst case is not an issue here: A worst case always exists. This is so because for any decision $q \in Q$ and any $\alpha \geq 0$, only the following situations arise, and in each situation there is at least one worst case, namely one worst u in $U(\alpha,\tilde{u})$:

- **Situation 1:** $r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})$.
  All the values of u in $U(\alpha,\tilde{u})$ are equally good (and bad) as they all satisfy the performance constraint. Hence, all the elements of $U(\alpha,\tilde{u})$ are worst cases and best cases.
- **Situation 2:** $r_c > r(q,u), \forall u \in U(\alpha,\tilde{u})$.
  All the values of u in $U(\alpha,\tilde{u})$ are equally bad (and good) as they all violate the performance constraint. Hence, all the elements of $U(\alpha,\tilde{u})$ are worst cases and best cases.
- **Situations 3:** $r_c \leq r(q,u)$ for some, but not all, $u \in U(\alpha,\tilde{u})$.
  All the elements of $U(\alpha,\tilde{u})$ that satisfy the constraint are best cases, and all elements of $U(\alpha,\tilde{u})$ that violate this constraint are worst cases.

Incidentally, the same applies to a **global** worst-case analysis of the constraint over the entire uncertainty space $\mathscr{U}$:

- **Situation 1:** $r_c \leq r(q,u), \forall u \in \mathscr{U}$.
  All the values of u in $\mathscr{U}$ are equally good (and bad) as they all satisfy the performance constraint. Hence, all the elements of $\mathscr{U}$ are worst cases and best cases. In this situation, decision q is super-robust with respect to the performance constraint.
- **Situation 2:** $r_c > r(q,u), \forall u \in \mathscr{U}$.
  All the values of u in $\mathscr{U}$ are equally bad (and good) as they all violate the performance constraint. Hence, all the elements of $\mathscr{U}$ are worst cases and best cases.
- **Situations 3:** $r_c \leq r(q,u)$ for some, but not all, $u \in \mathscr{U}$.
  All the elements of $\mathscr{U}$ that satisfy the constraint are best cases, and all elements of $\mathscr{U}$ that violate this constraint are worst case. In this situation, decision q is super-fragile.

In short, in the framework of info-gap's robustness analysis, the existence of a worst-case is not an issue because the worst-case analysis is conducted with respect to a performance **constraint.** Hence, a worst case exists even if the uncertainty space is unbounded. On top of this, the worst-case analysis is **local** because it is conducted over neighborhoods around $\tilde{u}$, one neighborhood at a time.

In sum, the assertions that info-gap's robustness analysis is not a worst-case analysis, e.g. Ben-Haim (2001, 2005, 2006, 2007, 2010, 2012a), demonstrate a failure to appreciate that info-gap's worst-case robustness analysis is **not a global** worst-case analysis that is conducted over the entire uncertainty space $\mathscr{U}$, but rather a **local** worst-case robustness analysis that is conducted on the (bounded) neighborhoods $U(\alpha,\tilde{u}), 0 \leq \alpha \leq \hat{\alpha}(q,\tilde{u})$, one neighborhood at a time. But more importantly, these assertions demonstrate a failure to appreciate that because the worst-case analysis is driven by a performance **constraint,** the existence of a worst case is a not an issue.

7 The maximin connection

The literature on info-gsp decision theory, e.g. Ben-Haim (2001, 2005, 2007, 2006, 2010, 2012a), is adamant that info-gap's robust-satisficing analysis is not a minimax/maximin analysis. Thus, consider these statements:

> The info-gap robustness function evaluates the greatest tolerable Knightian uncertainty. However, the info-gap robustness is not a minimax analysis (we do not ameliorate a worst case) since, in an info-gap model of uncertainty, there is no worst case: any given horizon of uncertainty is less severe than all greater horizons of uncertainty.
>
> Ben-Haim (2005, pp. 400-401)

> Optimization of the robustness in eq. (3.172) is emphatically not a worst-case analysis. In classical worst-case min-max analysis the decision maker minimizes the impact of the maximally damaging case. But an info-gap model of uncertainty is an unbounded family of nested sets: $U(\alpha, \tilde{u})$, for all $\alpha \geq 0$. Consequently, there usually is no worst case: any adverse occurrence is less damaging than some other more extreme event occurring at a larger value of α.
>
> Ben-Haim (2006, p. 101)

> Info-gap theory is not a worst-case analysis. While there may be a worst case, one cannot know what it is and one should not base one's policy upon guesses of what it might be. Info-gap theory is related to robust-control and min-max methods, but nonetheless different from them. The strategy advocated here is not the amelioration of purportedly worst cases.
>
> Ben-Haim (2010, p. 9)

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst. **The min-max concept responds to severe uncertainty that nonetheless can be bounded.** The info-gap concept responds to severe uncertainty that is **unbounded** or whose **bound is unknown.**
>
> Ben-Haim (2012a, p. 1644)

As indicated in the preceding section, the technical rationale that is put forward to justify these and similar claims made elsewhere, is that for a worst-case analysis to be applicable, worst outcomes must exist, and for this to be the case the domain of the analysis must be **bounded**. But since the info-gap uncertainty is unbounded, info-gap's robustness analysis cannot possibly be a worst-case analysis, hence it cannot possibly be a minimax/maximin analysis.

The assertions on the relation between info-gap's robustness analysis and maximin analysis in the literature on info-gap decision theory, e.g. Ben-Haim (2001, 2006, 2005, 2010, 2012a), exhibit a lack of familiarity with the large and important class of maximin models where the existence of worst outcomes is not an issue irrespective of whether the domain of the analysis is bounded or unbounded.

This is the class of maximin models that seek robustness **only with respect to performance constraints,** and not with respect to payoffs/rewards. In such cases the payoff/reward function $f = f(x, s)$ in (16) is independent of the state variable s, hence $f(x, s) = g(x), \forall x \in X, s \in S(x)$ where g is a real-valued function on X. These maximin models have the following generic form

$$z^* := \max_{x \in X} \min_{s \in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\}\,|_{f(x,s)=g(x)} \tag{30}$$

$$= \max_{x \in X} \min_{s \in S(x)} \{g(x) : con(x,s), \forall s \in S(x)\} \tag{31}$$

$$= \max_{x \in X} \{g(x) : con(x,s), \forall s \in S(x)\}. \tag{32}$$

As shown in the proof of Theorem 4.3, info-gap's robustness model and info-gap's robust-satisficing decision model are subsumed by this important class of maximin models. The following question is therefore inevitable:

- How is it that peer-reviewed journals continue to accept erroneous arguments in support of the claims that info-gap's robustness analysis is not a worst-case analysis and that info-gap's robustness model and info-gap's robust-satisficing decision model are not maximin models?

8 The radius of stability connection

Perhaps the greatest offender in the rhetoric on info-gap decision theory, in the literature on this theory, is the lack of all reference to the fact that the concept *info-gap robustness* is a re-invention of the concept *radius of stability* (circa 1960). So, keeping in mind the discussion in section 4.2, let us "expose" what seems to be a "top secret" in the info-gap literature by juxtaposing the two formal models that (in this discussion) give expression to these concepts:

Info-gap's robustness model	Radius of stability model	
$\hat{\alpha}(q,\tilde{u}) := \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	$\hat{\rho}(q,\tilde{u}) := \max_{\alpha \geq 0} \{\alpha : con(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(33)

recalling that $con(x,u)$ denotes the list of performance constraint imposed on the (q,u) pairs in the radius of stability analysis.

By inspection, info-gap's robustness model is a simple radius of stability model corresponding to the case where the list of constraints, $con(q,u)$, consists of one constraint, namely the constraint $r_c \leq r(q,u)$ associated with info-gap's robustness analysis. The proof of Theorem 4.2 is based on this simple observation.

Given these hard fact, the following questions are inevitable:

- How is it that info-gap scholars avoid mentioning that info-gap's robustness is a reinvention of a generic measure of local robustness that is known universally as *radius of stability* (circa 1960)?
- How is it that *Risk Analysis* referees, who should be well-informed on this subject, do not require that articles on info-gap decision theory come clean on the intimate relationship between info-gap robustness and radius of stability robustness radius, or at least refer to the vast literature on radius of stability models and their applications?

9 Local vs global robustness

Many info-gap publications erroneously treat the info-gap robustness of a decision as a measure of global robustness. In these publications, info-gap robustness is mistakenly associated with a measure of robustness that is based on the "size" or "volume" of the set of acceptable values of the parameter of interest (see section 4.1).

But, as we saw in section 4.1, there is a fundamental difference between the info-gap robustness of decision q and the size-robustness of decision q. Because, while the former is determined by the size of the largest **neighborhood** around $\tilde{u}$ that is contained in $A(q)$, the latter is determined by the size of the largest **subset** of $\mathscr{U}$ that is contained in $A(q)$, which is of course the seize of $A(q)$ itself. The implication of this fact is that while info-gap robustness is a measure of **local** robustness, the size-robustness is a measure of **global** robustness. This fact is practically "broadcast" by the respective mathematical formulations of the two models:

Info-gap's robustness model	Domain/volume/size model
$\max_{\alpha\ge 0} \{\alpha : r_c \le r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	$\max_{V\subseteq \mathscr{U}} \{SIZE(V) : r_c \le r(q,u), \forall u \in V\}$

(34)

To reiterate a point made above. An important consequence of the fact that info-gap's robustness is local is that, much as a decision's info-gap robustness, say that of decision q', is larger than the info-gap robustness of decision q'', this does not imply that $Size(A(q')) > Size(A(q''))$, and vice versa. This is illustrated in Figure 1.

And yet, info-gap scholars fail to distinguish between these two fundamentally different notions of robustness. And, to see what I have in mind, consider the following statements:

> Info-gap analysis allows the decision maker to identify solutions that perform satisfactorily well under the widest possible range of conditions.
>
> Hall and Ben-Haim (2007, p. 7)

> When operating under severe uncertainty, a decision which is guaranteed to achieve an acceptable outcome throughout a large range of uncertain realizations is preferred to a decision which can fail to achieve an acceptable outcome even under small error.
>
> Ben-Haim (2010, p. 8)

> The maximizer of utility seeks the answer to a single question: which option provides the highest subjective expected utility. The robust satisficer answers two questions: first, what will be a "good enough" or satisfactory outcome; and second, of the options that will produce a good enough outcome, which one will do so under the widest range of possible future states of the world.
>
> Schwartz, Ben-Haim and Dasco (2011, p. 213)

> It asks, instead, "What kind of return do we want in the coming year, say, in order to compare favorably with the competition? And what strategy will get us that return under the widest array of circumstances?"
>
> Schwartz, Ben-Haim and Dasco (2011, p. 220)

> For an individual who recognizes the costliness of decision making, and who identifies adequate (as opposed to extreme) gains that must be attained, a satisficing approach will achieve those gains for the widest range of contingencies.
>
> Schwartz, Ben-Haim and Dasco (2011, p. 223)

> Decisions that cause the system to exceed the performance criterion over a wide range of uncertainty are said to be more "robust" or "immune to failure" (Ben-Haim 2006).
>
> van der Burg and Tyre (2011, 304)

It would seem that this misguided rhetoric, which utterly misrepresents the facts about info-gap's robustness model, hence info-gap robustness, is an inevitable consequence of the misconceptions created by the effusive rhetoric in the literature about info-gap decision theory's purported extraordinary ability to deal with an extreme uncertainty that *inter alia* is manifested in an unbounded uncertainty space. Because, if info-gap decision theory, as trumpeted in this literature, has the capabilities to handle an unbounded, non-probabilistic uncertainty, then the next step would be to attribute to its robustness analysis the capability to determine the widest range of acceptable outcomes.

The truth is, of course, that this characterization of info-gap robustness flies in the face of info-gap decision theory's very definition of robustness.

The important point to note about such statements is that their characterization of info-gap robustness sweeps under the carpet the central role that the point estimate $\tilde{u}$ plays in an info-gap robustness analysis, which as we saw above, therefore renders the analysis and its results *inherently local*. This means that info-gap's robustness analysis does not identify a decision that perform satisfactorily well under the widest possible range of conditions. Rather, it seeks a decision that performs satisfactorily over the largest *neighborhood* $U(\alpha,\tilde{u})$ *around the nominal point* $\tilde{u}$.

That said, consider this assertion which seems to present a rebuttal to the claim that info-gap's robustness analysis is *inherently local* (emphasis added):

> For small α, searching set $U(\alpha,\tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha,\tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. **The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.**
>
> Hall et al. (2012, p. 1662)

This claim attests to an astounding misconception about the status of the parameter α in the definition of $\hat{\alpha}(q,\tilde{u})$, hence the role of the parameter α in info-gap's robustness analysis which is dictated by the following definition of info-gap robustness:

$$\hat{\alpha}(q|\tilde{u},r_c) := \max_{\alpha\geq 0}\{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} \tag{35}$$

where the notation $\hat{\alpha}(q|\tilde{u},r_c)$ is deployed to stress that the definition of the info-gap robustness of decision q posits that $\tilde{u}$ and r_c are given, fixed values.

A cursory examination of (35) is all it take to see how misguided Hall et al's (2012) statement is. Because, as this definition of info-gap robustness unambiguously indicates, the analyst has no control over the value of α. Indeed, what this definition makes clear is that, α is a "decision" variable whose optimal value is determined by the $\max$ operation. This means that it is not up to the analysts to decide on a whim to determine the info-gap's robustness of decision q for say, $\alpha = 34.89$.

One suspects, though, that Hall et al. (2012) might be alluding to the fact that the value of $\hat{\alpha}(q|\tilde{u}, r_c)$ can be changed (increased) indirectly by changing (decreasing) the value of the critical performance level r_c, because the smaller the value of r_c the less restrictive the performance constraint $r_c \leq r(q, u)$, hence the larger the value of $\hat{\alpha}(q|\tilde{u}, r_c)$. Hence, $\hat{\alpha}(q, |\tilde{u}, r_c)$ is non-increasing with r_c.

But consider the implications of such a proposition. On this reasoning, "at the limit", namely as $\hat{\alpha}(q|\tilde{u}, r_c)$ approaches ∞, where $U(\alpha, \tilde{u})$ becomes as large as $\mathscr{U}$, the value of r_c becomes so small, that the constraint $r_c \leq r(q, u)$ is practically rendered **redundant,** to the effect that robustness is not an issue. That is, for such a small value of r_c, decision q satisfies the performance constraint $r_c \leq r(q, u)$ for all $u \in \mathscr{U}$, which means that determining the robustness of decision q is not an issue to begin with, because total robustness over $\mathscr{U}$ is trivially obvious.

In a word, according to Hall et al. (2012, p. 1662), for info-gap's local robustness analysis to become global, the analyst must decrease the the value of r_c so as to force the performance constraint to become redundant! A "novel" and "informative" analysis indeed!

Finally, it seems that, Hall et al. (2102) apparently confuse two facts about info-gap decision theory:

- The fact that info-gap's robustness model seeks robustness against small perturbations in a nominal value of parameter of interest, which renders it a model of **local** robustness.
- The fact that one can systematically vary the value of r_c so as to determine the effect of a more/less restrictive performance constraint on the info-gap robustness of decision q, namely on $\hat{\alpha}(q|\tilde{u}, r_c)$.

A more detailed discussion on Hall et al. (2012) can be found in Sniedovich (2012e).

Next consider this statement:

> If the robustness is not large, and especially if the robustness is small, then confidence is not warranted. If the robustness is small then confidence is warranted only "locally," near the models, while if the robustness is large then confidence is warranted over a wide domain of deviation from the models. Info-gap theory uses the analyst's models, but this does not make it a "local" theory of robustness.
>
> Ben-Haim (2012a, p. 1644)

The magnitude of misconception that is revealed by this statement is truly staggering. It would seem therefore that info-gap scholars need reminding that the question whether robustness is *global* or *local* is not decided by whether the robustness yielded is *small* or *large*, but *by the definition of robustness* under consideration. This means that the fact that the info-gap robustness of some decisions turns out to be large does not make info-gap decision theory a theory of global robustness.

The best way to clarify this point is to draw an analogy with *optimization theory.* As we all know, the fact that the optimal solution obtained by an optimization method turns out to be a global optimum does not imply that the method in question is a global optimization method. For, as we all know, under certain conditions, the solutions yielded by methods of local optimization are global optima. It goes without saying, though, that these conditions need to be spelled out!

Which brings us to the claim that "... If the robustness is small then confidence is warranted only "locally," near the models... ". Ben-Haim (2012a) apparently does not realize that this claim in fact reinforces the proposition that info-gap decision theory is "... a "local" theory of robustness". Because what this claim confirms is that when the info-gap robustness of a decision is small, info-gap decision theory rules that confidence in this decision's performance is warranted only **locally**, namely near the point estimate $\tilde{u}$. But as I have shown in Figure 1, this ruling itself is most definitely unwarranted, because a decision may well be *locally* robust but *globally* fragile, and vice versa. In a word, the fact that info-gap decision theory makes this ruling on grounds of a *local analysis,* corroborates the claim that it is "... a "local" theory of robustness."

The same argument applies to the claim that "... if the robustness is large then confidence is warranted over a wide domain of deviation from the models ... " Because, it is elementary to construct examples where the info-gap robustness of decision q'' at $\tilde{u}$ is much larger than the info-gap robustness of decision q' at $\tilde{u}$, yet decision q' is far more robust than decision q'' according to measures of global robustness. For instance, consider the case where decision q' satisfies the performance constraint for all $u \in \mathscr{U}$ except for a single $u' \in \mathscr{U}$ which is very close to $\tilde{u}$. Then, the info-gap robustness of decision q' would be very small even though q' is extremely robust globally over $\mathscr{U}$.

As for the punch line " ... Info-gap theory uses the analyst's models, but this does not make it a "local" theory of robustness ... ", info-gap scholars should be reminded that info-gap's robustness analysis stands or falls with "the analyst's model", namely with the value of $\tilde{u}$. After all, info-gap robustness is defined as the measure of the smallest critical deviation from "the analyst's model". Therefore the value of $\tilde{u}$ is critical in determining the robustness of decisions, hence their ranking: a change in the value of $\tilde{u}$ may change their robustness, hence their ranking. This means that info-gap's robust-satisficing approach to severe uncertainty, which comes down to a ranking of decisions according to their info-gap's robustness, equally stands or falls with "the analyst's model". All this goes to show that info-gap decision theory is "... a "local" theory of robustness ... " *par excellence*.

Theorem 3.1, the *No Man's Land* property, and Figure 3 amply demonstrate this fact. More on this subject can be found in Sniedovich (2012e).

10 The robust optimization connection

One of the worst offenders in info-gap decision theory's rhetoric is the rhetoric on "optimizing vs satisficing" which presumably explains the rationale behind this theory's supposed innovative approach to robust decision making under severe uncertainty, namely info-gap's "robust satisficing" approach. Thus, a recurring theme in this rhetoric is that optimal solutions often lack robustness to variations in the values of the optimization model's parameters, therefore, under uncertainty "satisficing" is more fitting than "optimizing". For instance:

> Another immediate result is that the robustness of the optimal result—the maximal reward under our best estimate $\tilde{u}$—is not robust. In fact, it has zero robustness, meaning that a slight deviation from our estimation $\tilde{u}$ may prevent us from meeting the requirement r_c.
>
> Davidovitch et al. (2009, p. 2788)

> Decision makers often face severe uncertainties. This has profound implications for any attempt to optimize the outcome of their decisions. In this essay, we first discuss a paradox from financial

> economics that belies the cardinality of performance optimization by economic agents. We contrast performance optimization with a strategy of robustly achieving critical goals.
>
> Ben-Haim (2012, p. 1326)

> Accepting a suboptimal but adequate investment is an example of what Simon called satisficing, and it also motivates the idea of robustness.
>
> Ben-Haim (2012, p. 1327)

> In other words, satisficing is more robust to uncertainty than optimizing. Hence, this strategy is called robust-satisficing. If satisficing—rather than maximizing—is in some sense a better bet, then it will tend to persist under uncertain competition.
>
> Ben-Haim (2012, pp. 1327-1328)

> *3.3.4. Summing Up: Robust-Satisficing and the Innovation Dilemma*
>
> An innovation dilemma arises when one must choose between a seemingly better (innovative) option that is more uncertain, and a more thoroughly understood but possibly less attractive (tried and true) option. The first step in balancing between uncertainty and decisiveness is to identify critical or necessary outcomes. Then, in a range of situations, a satisficing decision strategy that is maximally robust to our ignorance is a better bet than other strategies, for instance, outcome optimization, as we have explained earlier.
>
> Ben-Haim (2012a, p. 1643)

Reading this rhetoric, one wonders at the referees, who should have been familiar with the thriving field of *robust optimization* (Kouvelis and Yu 1997, Ben-Tal et al. 2009), which was established some 40 years ago to deal precisely with situations where optimal solutions are required to be robust against variations in the values of the optimization model's parameters. Surely, the referees who accepted the misleading rhetoric on 'satisficing vs optimising' should have known that Simon's "satisficing" approach has long been incorporated in optimization models:

> Chance constrained programming admits random data variations and permits constraint violations up to specified probability limits. Different kinds of decision rules and optimizing objectives may be used so that, under certain conditions, a programming problem (not necessarily linear) can be achieved that is deterministic—in that all random elements have been eliminated. Existence of such 'deterministic equivalents' in the form of specified convex programming problems is here established for a general class of linear decision rules under the following 3 classes of objectives (1) maximum expected value ('E model'), (2) minimum variance ('V model'), and (3) maximum probability ('P model'). Various explanations and interpretations of these results are supplied along with other aspects of chance constrained programming For example, the 'P model' is interpreted so that H A SIMON'S suggestions for 'satisficing' can be studied relative to more traditional optimizing objectives associated with 'E' and 'V model' variants.
>
> Charnes and Cooper (1963, p. 18)
>
> A. Charnes, A., and W. W. Cooper, W.W. (1963) Deterministic equivalents for optimizing and satisficing under chance constraints. *Operations Research,* 11(1), 18-39.

Inexplicably and inexcusably, there is nothing in the three primary texts on info-gap decision theory (Ben-Haim 2001, 2006, 2010) that even hints at the existence of the field of *robust optimization.* Worse, there is nothing in these texts to so much as suggest that info-gap's so-called "robust satisficing" approach is in fact a simple (indeed, naive) *robust optimization* approach.

The same is true of the article *Doing Our Best: Optimization and the Management of Risk*, that was published recently in *Risk Analysis*, whose abstract reads as follows:

> Tools and concepts of optimization are widespread in decision-making, design, and planning. There is a moral imperative to "do our best." Optimization underlies theories in physics and biology, and economic theories often presume that economic agents are optimizers. We argue that in decisions under uncertainty, what should be optimized is robustness rather than performance. We discuss the equity premium puzzle from financial economics, and explain that the puzzle can be resolved by using the strategy of satisficing rather than optimizing. We discuss design of critical technological infrastructure, showing that satisficing of performance requirements—rather than optimizing them—is a preferable design concept. We explore the need for disaster recovery capability and its methodological dilemma. The disparate domains—economics and engineering—illuminate different aspects of the challenge of uncertainty and of the significance of robust-satisficing.
>
> Ben-Haim (2012, p. 1326)

The fact that not a single reference is made in this article to the field of *robust optimization* is remarkable, considering that it appears alongside Cox's article (2012) which lists *robust optimization* as one of 10 tools available for the management of robustness under severe uncertainty. The omission of *robust optimization* from Ben-Haim (2012, other) is especially glaring in view of the following lists:

> Many other concepts of robustness are to be found, including P-boxes for managing uncertainty in probability distributions in many applications,(19−22) imprecise probabilities,(23,24) info-gap theory,(10) and others.
>
> Ben-Haim (2012, p. 1329)

> We have robust statistics,(27) robust control,(28) robust decision making,(29) robust flexibility,(30) robust economics,(31) info-gap robustness,(18) and many more tools.
>
> Ben-Haim (2012,a p. 1640)

One therefore wonders: is the systematic absence of all reference to *robust optimization,* and of course to the obvious kinship of info-gap's robust-satisficing approach to *robust optimization* deliberate? Be it as it may, the far more important question is this:

- How is it that *Risk Analysis* referees did not insist that Ben-Haim (2012, 2012a) discuss the kinship between robust-satisficing and robust optimization?

The fact that info-gap's robust satisfying paradigm is a simple, more accurately simplistic, indeed naive, instance of *robust optimization* is explained in detail in: *Risk Analysis 101: Robust-Optimization—the elephant in the robust-satisficing room* (Sniedovich 2012c). As I show in this article , far from being a step forward, Ben-Haim's (2012) proposal to adopt info-gap's robust-satisficing approach to severe uncertainty is in fact a step backward to the early days of *robust optimization* of the 1970's. Namely, it is a step backwards to the era predating the latest developments in this field. But more than this, as I also explain in this article and elsewhere (e.g. Sniedovich 2007, 2010, 2012, 2012a), info-gap's so-called robust satisficing approach, lacks the **requisite technical wherewithal** to deal with the type of severe uncertainty stipulated by info-gap decision theory.

Remark

It is interesting to note that Ben-Haim (2012) discusses in some detail the contribution of Hansen and Sargent (2007) to the introduction of robustness tools in economics:

> Hansen and Sargent have pioneered the introduction of robustness tools in economics. In their recent book,(18) they quantify model misspecification by taking "a given approximating model and surrounding it with a set of unknown possible data generating processes, one unknown element of which is the true process ... Our decision maker confronts model misspecification by seeking a decision rule that will work well across a set of models for which" the relative entropy is bounded.
>
> ⋮
>
> They then "maximize [an] intertemporal objective over decision rules when a hypothetical malevolent nature minimizes that same objective ... That is, we use a max-min decision rule."
>
> Ben-Haim (2012, p. 1329)

And yet, there is not a single reference to Hansen and Sargent's pioneering work on robust decision-making in economics in the book: *Info-gap Economics: An Operational Introduction* (Ben-Haim 2010).

I mention this fact to point out that this seems to be of a piece with the effort in the literature on info-gap decision theory to present the theory as offering a "novel", "radically different" approach to decision-making under severe uncertainty.

The discussion on "deterministic equivalents" in the next section is another illustration of this effort to portray info-gap decision theory as "radically different".

11 The deterministic equivalents connection

One of the boldest claims in the literature on info-gap decision theory is that this theory, which is trumpeted to be **non-probabilistic** and likelihood-free, can yield robust-satisficing decisions that **maximize the probability of success/survival,** namely maximize the probability that the performance constraint $r_c \leq r(q,u)$ is satisfied. Thus:

> Is a robust strategy a good probabilistic bet for achieving acceptable or desired outcomes? The question is important because likelihood of success is desirable. The question is difficult because most concepts of robustness are nonprobabilistic. The surprising answer is that even nonprobabilistic concepts of robustness often provide the best probabilistic bet. We touch on the explanation here and return to it in Section 3.3.
>
> Ben-Haim (2012, p. 1641)

> Our most general result—Proposition 1—assumes that the probability distribution and the info-gap model are coherent, as specified in Definition 1. Coherence entails weak informational overlap between the probability distribution and the info-gap model. We have shown that coherence holds in many situations, as seen in our examples and elsewhere (Ben-Haim 2011).
>
> Ben-Haim (2013, p. 12)

The objective of the discussion in this section is to set the record straight on this claim by bringing to light a number of hard facts attesting that:

- This ability is restricted to trivially simple, some might say naive, situations.
- In these trivially simple situations, the use of info-gap decision theory is not only uncalled for, it is in fact counterproductive.
- The proposition to have non-probabilistic optimization models act as "proxies" for probabilistic optimization models is longstanding, and it has been applied extensively for many years now in various fields.
- Unsurprisingly, contrary to Ben-Haim's (2013) assertions, this idea does indeed fall back on "probabilistic information".

I take up the last point first.

To this end consider the following statement (emphasis added):

> We present three propositions, based on info-gap decision theory (Ben-Haim 2006), that identify conditions under which the probability of success is maximised by an agent who robustly satisfices the outcome **without using probabilistic information.**
>
> Ben-Haim (2013, p. 1)

Reading this statement (and similar ones in the info-gap literature) you would think that the formulation of non-probabilistic optimization models as equivalents of probabilistic optimization models is based on alchemy or witchcraft. Doesn't it stand to reason that, the formulation of such models would be rooted in principles deriving from the . . . **axioms of probability theory**?

My point is then that contrary to the misleading rhetoric in Ben-Haim (2013), one would not be able to formulate the three propositions in question without "probabilistic information", namely without presupposing the following fundamental properties of **commutative probability distribution functions:**

$$a < b \longrightarrow F_Y(a) \leq F_Y(b) \tag{36}$$

where Y denotes a *random variable*, F_Y denotes the cumulative probability distribution function of Y, so by definition, $F_Y(a)$ denotes the probability of the event "$Y \leq a$". To simplify the notation, let

$$P(Y \leq a) := F_Y(a)\ ,\ a \in \mathbb{R} := (-\infty, \infty) \tag{37}$$

that is, let $P(Y \leq a)$ denote the probability of the event "$Y \leq a$".

Observe that in the context of (36)-(37), the information that "a is smaller than b" is most definitely "probabilistic". To emphasize this point, I re-write (36) as follows:

$$a < b \longrightarrow P(Y \leq a) \leq P(Y \leq b). \tag{38}$$

As indicated above, the concept of "deterministic equivalents", namely the idea of non-probabilistic models acting as equivalents (in some sense) of probabilistic models figures prominently in fields such as *operations research and stochastic optimization* (see the above quote from Charnes et al. 1963). For our purposes it suffices to note this elementary pair:

Probabilistic Problem	Deterministic Equivalent
$\max\limits_{x\in X} P(Y \leq h(x))$	$\max\limits_{x\in X} h(x)$

(39)

Since $P(Y \leq a)$ is non-decreasing with a, it follows that any optimal solution to the *Deterministic Equivalent* is also optimal with respect to the *Probabilistic Problem.* This means that we can maximize $P(Y \leq h(x))$ over $x \in X$ by maximizing $h(x)$ over $x \in X$, observing that the latter does not require knowledge of the probability distribution function of Y.

Now, in the context of info-gap decision theory, the performance constraint is $r_c \leq r(q, u)$, where u represents the uncertainty parameter. Hence the probabilistic robustness problem is as follows:

$$p^* := \max_{q\in Q} P(r_c \leq r(q, W)) \tag{40}$$

where W denotes the random variable governing the values of u.

So, translating the above simple scheme into the info-gap decision theory framework, the performance constraint $r_c \leq r(q, u)$ is transformed into $t(u) \leq h(q)$, so that we have:

$$\forall (q, u) \in Q \times \mathscr{U} : \ \ r_c \leq r(q, u) \longleftrightarrow t(u) \leq h(q) \tag{41}$$

where formally t is a real-valued function on $\mathscr{U}$ and h is a real-valued function on Q. This will yield

$$P(r_c \leq r(q, W)) = P(t(W) \leq h(q)). \tag{42}$$

So, we can let $Y := t(W)$, in which case:

Probabilistic Problem	Equivalent Probabilistic Problem	Deterministic Equivalent
$\max\limits_{q\in Q} P(r_c \leq r(q, u))$	$\max\limits_{q\in Q} P(t(W) \leq h(q))$	$\max\limits_{q\in Q} h(q)$

(43)

The following example illustrates a simple application of such a scheme.

Illustrative Example

Consider the case where $Q = [0, \overline{q})$, $\mathscr{U} = \mathbb{R}$, and the performance constraint is as follows

$$q + (\overline{q} - q)u \geq c \tag{44}$$

where c is a given numeric scalar. By inspection, we can rewrite this performance constraint as follows:

$$-u \leq \frac{q-c}{\overline{q}-q} \ . \tag{45}$$

Hence, we can let $t(u) = -u$ and $h(x) = \dfrac{q-c}{100-q}$, so that

Probabilistic Problem	Equivalent Probabilistic Problem	Deterministic Equivalent
$\max\limits_{q\in Q} \ P\left(q + (\overline{q}-q)u \geq c\right)$	$\max\limits_{q\in Q} \ P\left(-u \leq \dfrac{q-c}{\overline{q}-q}\right)$	$\max\limits_{q\in Q} \ \dfrac{q-c}{\overline{q}-q}$

(46)

This illustrative example raises three obvious questions:

- What has the above analysis got to do with info-gap decision theory?
- Is the above illustrative example representative of the probabilistic problems tackled by info-gap decision theory?
- Given the ease with which the *Deterministic Equivalent* can be derived in this case, what is to be gained by using info-gap decision theory for this purpose?

To answer the first question I must first note that in info-gap decision theory's jargon, the conditions under which info-gap's robustness model is claimed to "maximize the probability that the performance constraint is satisfied", are spelled out by "proxy theorems". Now, the rhetoric about these theorems gives the impression that they derive directly from the axioms of info-gap decision theory itself. The fact of the matter is of course that, once the rhetoric on this matter, in publications such as Ben-Haim (2013), is peeled away, one reveals (as we saw above) that the formulation of "proxy theorems" is reliant in this effort on the axioms of probability theory.

The question is whether these so-called "proxy theorems" enable solving problems that are worth their salt! And the answer to this question answers the second question posed above. The illustrative example discussed above is indeed representative of the probabilistic problems tackled by info-gap decision theory via non-probabilistic robustness model. This example is the principal example examined in Ben-Haim (2013). The problem featured in Ben-Haim and Cogan (2011) in even simpler.

Which brings us to the third question posed above. Considering then that (so far as can be ascertained) all that info-gap decision theory can do is to take on probabilistic problems that have trivial deterministic equivalents, what merit, point or advantage, can there possibly be in using info-gap's robustness model to obtain a *Deterministic Equivalent* in such cases. The fact of the matter is that using info-gap's robustness model to this end is not only pointless it is counterproductive!

The trouble is, however, that Ben-Haim (2012a, 2013) and Ben-Haim and Cogan (2011) do not apprise the readers of these articles that the conditions imposed by info-gap's "proxy theorems" are so stringent that the problems that these theorems end applying to turn out to be extremely simple, in fact **trivially easy** to deal with "deterministically". And what is more, this hard fact is buried under the "mountain of mathematical analysis" that is requires to deduce these "proxy theorems", so that it is next to impossible to lift it out from under this mountain of mathematical arguments. However, those who are at home with the longstanding concept *Deterministic Equivalents,* can immediately tell that the non-probabilistic models that info-gap decision theory deploys to maximize probability of success/survival can be deduced directly indeed, **by inspection** from the performance constraint under consideration. In which case one can immediately see that the problems purportedly tackled by info-gap's non-probabilistic model can be solved **by inspection.** Meaning that formulating these problems in terms of info-gap's non-probabilistic model, or for that matter, any other model, is utterly uncalled for as this does nothing but unduly complicate probabilistic problems whose deterministic equivalents stare one in the face.

One of the most comprehensive examination of this topic, giving a picture of info-gap's "proxy theorems" that is far more accurate than that given by Ben-Haim (2012a, 2013) and Ben-Haim and Cogan (2011), can be found in Davidovitch (2009). Davidovitch (2009) draws a distinction between *strong proxy theorems* and *weak proxy theorems*. Members of the *strong* class do not impose any requirement on the probability/likelihood structure of the uncertainty space under consideration. Therefore,"strong proxy theorems" need to stipulate conditions guaranteeing that the decision selected by info-gap's robustness model maximizes the "probability of success", regardless of the underlying (completely unknown) probability/likelihood structure.

In contrast, members of the *weak* class impose some "coherency" conditions on the probability/likelihood structure. These conditions are designed to ensure that the probability/likelihood structure associated with the uncertainty space "mimic" the implicit "distance" function employed by info-gap's model of uncertainty to create the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ around $\tilde{u}$.

It is important to take note that Davidovitch's (2009) overall conclusion is that proxy theorems are expected to be "very rare" (emphasis added):

> We have shown that the definition of strong proxy theorems discussed by Ben-Haim (2007), is **very restrictive,** and that when the uncertainty is multi-dimensional, strong proxy theorems are expected to be **very rare.** Then we shall prove that even this weaker definition **does not hold** for a **wide family of common problems.**
>
> Davidovitch (2009, p. 137)

The reason that "…even this weaker definition **does not hold** for a **wide family of common problems.**" is discussed in detail in Sniedovich (2013). In a nutshell, the idea of seeking to solve a non-probabilistic problem via its *deterministic equivalent,* in the case of info-gap decision theory, implies linking a model pursuing a robustness $\hat{\alpha}(q, \tilde{u})$ that is **local** in nature, with a model pursuing a probabilistic robustness $P(r_c \leq r(q, u))$ that is **global** in nature. Therefore, the principal role of the conditions imposed by info-gap's "proxy theorems" is to ensure that these two measures of robustness yield the same ranking of decisions. Unsurprisingly, to create this nexus one needs to impose exacting conditions on the performance constraint $r_c \leq r(q, u)$. The point is, though, that these conditions simplify the performance constraint itself to such an extent that one ends with a constraint of the form $t(u) \leq h(x)$, namely a *deterministic equivalent* that does not require info-gap decision theory's heavy guns to begin with.

To illustrate.

Theorem 4. *Assume that the performance levels $r(q, u), q \in Q, u \in \mathscr{U}$, can be decomposed so that*

- $r(q, u) = F(q, t(u))$ *where t is a real-valued function on $\mathscr{U}$ and F is a real-valued function on $Q \times \mathbb{R}$.*
- *For any given $q \in Q$ the value of $F(q, t(u))$ is non-increasing with $t(u)$.*

- *For every $q \in Q$ the value of*

$$h(q) := \max_{u \in \mathscr{U}} \{t(u) : r_c \leq F(q, t(u))\} \tag{47}$$

is attained.

Then $r_c \leq r(q,u) \longleftrightarrow t(u) \leq h(q)$ *and therefore* $\max_{q \in Q} h(q)$ *is a* DETERMINISTIC EQUIVALENT *to* $\max_{q \in Q} P(r_c \leq r(q,u))$.

Proof. This follows immediately from the definition of $h(q)$ in (47) and the fact that $F(q, t(u))$ is non-increasing with $t(u)$. **QED**

To repeat, **this has got nothing whatsoever to do with info-gap decision theory as such.** It has very much to do with properties of the performance levels $r(q,u), q \in Q, u \in \mathscr{U}$, and a fundamental property of **probability measures,** namely $a < b \longrightarrow P(Y \leq a) \leq P(Y \leq b)$.

12 Info-gap decision theory: some hard facts

The inference to be drawn from what we have seen so far is clear. Once the rhetoric in the literature on info-gap decision theory describing info-gap's robust-satisficing approach is peeled away and the focus is squarely on this theory's mathematical models and their mode of operation, the following hard facts are exposed:

- **Fact 1:** Both info-gap's robustness model and info-gap's robust-satisficing decision model, are simple **maximin models.**
- **Fact 2:** Info-gap's robustness model is a simple **radius of stability** model.
- **Fact 3:** Info-gap decision theory postulates that the uncertainty under consideration is **severe** in the sense that the uncertainty space $\mathscr{U}$ can be **vast** (e.g. unbounded), the point estimate $\tilde{u}$ can be just a **guess,** and the uncertainty is **non-probabilistic** and **likelihood-free.**
- **Fact 4:** The info-gap robustness of decision q is a measure of the decision's **local** robustness at $\tilde{u}$. Namely, it is the size of the **smallest perturbation** in the point estimate $\tilde{u}$ such that, if increased, it will cause a violation of the performance constraint $r_c \leq r(q,u)$ for some value of u determined by the increased perturbation in the value of $\tilde{u}$.
- **Fact 5:** Much as info-gap decision theory's definition of the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$, allows α to increase indefinitely, info-gap's robustness model dictates that in the analysis of the robustness of decision q at $\tilde{u}$, **the admissible value of α is bounded above by $\alpha' = \hat{\alpha}(q, \tilde{u}) + \varepsilon$,** hence effectively $\mathscr{U} = U(\alpha', \tilde{u})$.
- **Fact 6:** Thus, info-gap decision theory's definition of the robustness of decision q is **completely oblivious to** the decision's resilience/vulnerability to deviations in $\tilde{u}$ that are larger than $\hat{\alpha}(q, \tilde{u})$. Namely, it takes no account of values of u in the *No Man's Land* $NML(q, \tilde{u}, \varepsilon)$.
- **Fact 7:** Info-gap's robust-satisficing approach to non-probabilistic uncertainty is a very simple, some will argue naive, **robust-optimization approach.** However, its inherently **local** orientation, makes it

utterly unsuitable for the treatment of a severe uncertainty of the type stipulated by info-gap decision theory.

- **Fact 8:** Info-gap decision theory's ability to maximize the probability of success/survival derives from fundamental properties of **probability measures** and as such has absolutely nothing to do with info-gap decision theory itself. Furthermore, this "ability" is limited to extremely simple cases that fall under what are known in the fields of operations research and stochastic optimization as *deterministic equivalents,* which date back to the 1960s.

I hasten to add that my objective in bringing these facts to light is neither to find fault with the fact that info-gap decision theory's two core models are maximin models, nor with the fact that these models are based on a local measure of robustness. Obviously, many non-probabilistic models of robustness are maximin models and there are many applications where robustness against small perturbations in the nominal value of a parameter is precisely the type of robustness sought by analysts and decision-makers.

By the same token, there is nothing "wrong" with applying the concept "deterministic equivalent" in the framework of info-gap decision theory, despite the incongruity between the global orientation of the probabilistic robustness model and local orientation of info-gap's robustness model.

The trouble in info-gap decision theory lies elsewhere.

The trouble with info-gap decision theory is that its rhetoric persistently denies that its two core models are maximin models, or at best this rhetoric obfuscates on this matter. Likewise, the trouble with info-gap decision theory is that its implementation of these models amounts to a gross misapplication of Wald's maximin paradigm in that its application of this paradigm is utterly unsuitable for the purpose of seeking robustness to the **severe** uncertainty that the theory stipulates. This is so because info-gap decision theory's prescription for robustness employs these maximin models to seek a **local** robustness, namely a robustness in the **neighborhood** of the point estimate. This is a robustness that **disregards** the performance of decisions over large subsets of the uncertainty space under consideration.

Similarly, the trouble with info-gap decision theory is that its rhetoric about its ability to maximize the probability of success/survival is grossly misleading in that it conceals a great deal more than it reveals about this claimed ability.

The same applies to the misleading rhetoric on "robust-satisficing". Sounding as though the thriving field of robust optimization does not exist, it claims among other things, that optimization theory fails to provide the requisite means for obtaining optimal solutions that are robust against an uncertainty in the values of the optimization model's parameters.

My objective in this discussion was to make it clear that a correct assessment of info-gap decision theory requires an appreciation of the above basic hard facts.

13 Common sense

One need not be a professional risk analyst to see that the inherently local orientation of info-gap's robustness model is incompatible with the severity of the uncertainty that this theory claims to address, a "severe uncertainty", which one must remember, is clearly spelled out in Ben-haim (2001, 2006). This means that as a theory that, by virtue of its definition of robustness, pursues **local robustness,** info-gap decision theory lacks the technical wherewithal required for pursuing **global robustness** which is indeed demanded by such an uncertainty. The upshot of this is that in situations where the severity of the uncertainty is manifested in an unbounded uncertainty space, which, as pointed out by Ben-Haim (2001, 2006) in info-gap decision theory, is typically the case, info-gap's prescription for the management of severe uncertainty typically amounts to a prescription for "voodoo decision-making" Sniedovich (2010, 2012, 2012a). This is illustrated in Figure 3.

There are signs that some info-gap proponents realize that rhetoric alone cannot possibly reconcile this fundamental incompatibility. So, to overcome it, the following proposition was made (emphasis added):

> An assumption remains that values of u become **increasingly unlikely** as they diverge from $\tilde{u}$.
>
> Hall and Harvey (2009, p. 12)

This was an attempt to justify the local orientation of info-gap decision theory, namely its focusing on the point estimate $\tilde{u}$ and its making do with a robustness analysis conducted over a neighborhood around the point estimate $\tilde{u}$. The trouble with this *ad hoc* assumption, as explained in Sniedovich (2010, 2012), is that not only does it fly in the face of the proclaimed likelihood-free nature of info-gap decision theory's uncertainty model. It is not sufficiently strong to achieve that which it was designed to achieve, as on it own it does not guarantee that the true value of u is very likely to be in the immediate neighborhood of the estimate $\tilde{u}$. But what is more, it is straightforward to construct examples where this assumption is satisfied, yet the true value of u is very unlikely to be in the neighborhood of the estimate $\tilde{u}$.

In any case, while Hall and Harvy's (2009) attempt at fixing the incompatibility issue should be acknowledged, it must be noted that it is not mentioned at all in Hall et al.'s (2012) recent article. Why?

Finally consider the following eloquent, albeit diplomatic note on the incompatibility issue (emphasis added)[1]:

> If, for a given critical reward, the preferred management strategy varies with the initial estimate of an uncertain parameter, the analyst will be forced to consider the plausibility of different parameter values in order to identify a preferred strategy, particularly if the robustness of the alternative strategies are similar. **IGT, however, does not provide any estimate of the plausibility of different values, this issue is left with the analyst. Analysts who were attracted to IGT because they are very uncertain, and hence reluctant to specify a probability distribution for a model's parameters, may be disappointed to find that they need to specify the plausibility of possible parameter values in order to identify a robust management strategy.**
>
> Hayes (2011, p. 90)

The fundamental question that common sense alone demands an answer to is then this:

- How can one justify the claim that a non-probabilistic, likelihood-free theory, that prescribes a local robustness analysis in the neighborhood of a point estimate, is "a reliable" method for robust decision-making under severe uncertainty, given that the point estimate can be a wild guess and the uncertainty space is typically unbounded?!

[1] IGT = info-gap decision theory.

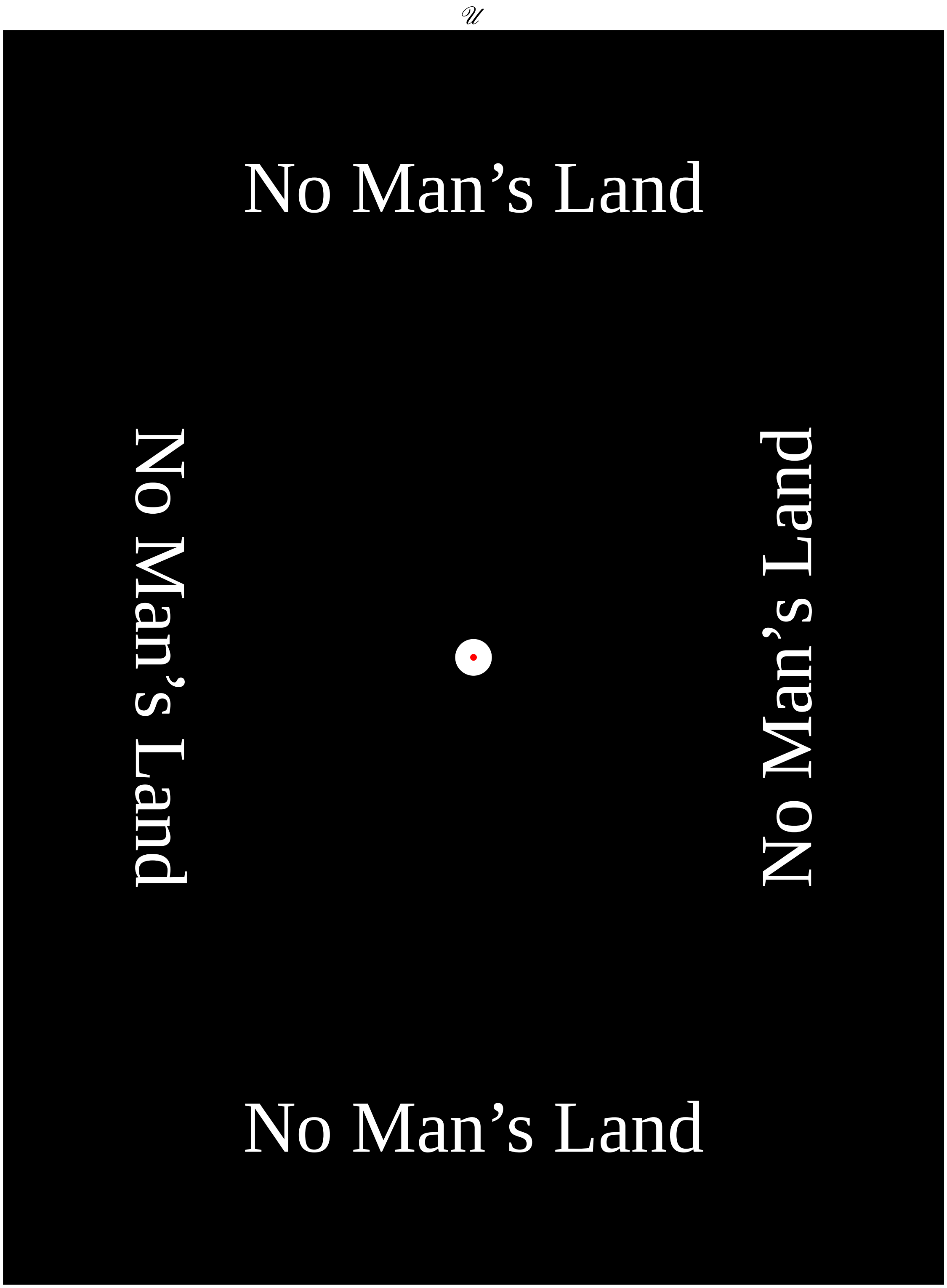

Figure 3: A profile of a voodoo decision theory. The uncertainty space $\mathscr{U}$ is vast (e.g. unbounded), and the domain of the robustness analysis is a small neighborhood around a wild guess of the true value of the parameter of interest, represented by the small white circle. The **No Man's Land** represents that part of the uncertainty space that is completely ignored by the robustness analysis. Since, according to Ben-Haim (2001, 2006, 2010), the uncertainty space of problems analyzed by info-gap decision theory is typically **unbounded,** it follows that the domain of info-gap robustness analysis is typically infinitesimally small compared to the uncertainty space under consideration.

The literature on info-gap decision theory does not address this common sense question.

Isn't it time that referees of risk analysis journals insisted that proponents of info-gap decision theory address this and other related fundamental questions? This is long overdue.

14 Summary and conclusions

In this article I explain, yet again, that there is a yawning gap in the info-gap literature between the rhetoric describing this theory, its mode of operation, its capabilities and its scope, and the hard facts attesting to what this theory really is and what its capabilities are. As we saw in the preceding discussions, contrary to this rhetoric, not only is this theory not radically different from mainstream theories, its two core models are simple ... maximin models. In fact, we saw that its robustness model is a re-invention (a "clone") of the famous *radius of stability model*.

We saw that info-gap's robust-satisficing approach to severe uncertainty is not a novel approach, but rather a simple, some would say naive, indeed simplistic *robust optimization* approach.

We saw that for all the rhetoric about it allowing the uncertainty space to be unbounded, info-gap decision theory prescribes a local robustness analysis, and as such it is utterly unsuitable for the treatment of a severe uncertainty that is manifested in an unbounded uncertainty space.

We saw that its concept of "proxy theorems" is a re-invention of the well-established concept *Deterministic Equivalents* and that, for the most part, the application of these theorems, for the purpose of "maximizing the probability of success without using probabilistic information", is not only pointless it is in fact counter-productive.

In short, we saw that the narrative in the literature on info-gap decision theory paints an utterly distorted picture of this theory, and of the state of the art in the broad area of decision under severe uncertainty.

And to sum it all up.

My experience of the last eight years has shown that it is apparently quite easy to be taken in by the narrative in the info-gap literature. I suspect that the nomenclature used by this theory is a major contributing factor to this theory's unwarranted appeal. Thus monikers like "info-gap", "robust-satisficer", "opportuneness model" and the hollow rhetoric spun around them, seem to be particularly attractive to scholars/analysts who are not at home in areas of expertise that have a direct bearing on info-gap decision theory, for instance, decision theory, optimization theory, notably robust optimization. Surprisingly, though, this narrative has also managed to mislead scholars/analysts with a firm background in quantitative methods who should have known better.

It is important to note that info-gap advocates are in the habit of making extravagant statements about this theory without bothering to offer the slightest technical proof to substantiate them. The claim: "...The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative ..." in Hall et al. (2012, p. 1662) is a perfect example of this practice.

It is important to appreciate, therefore, that the whole of info-gap decision theory boils down to two simple mathematical models whose prototype model was introduced several decades ago, and has since become staple fare in various disciplines: decision theory, engineering, economics etc. so that the body of literature about it is enormous. It is imperative therefore that the claims made by info-gap advocates about this theory, its capabilities, and its scope, be carefully checked, in the peer-reviewed process, by referees who are at home in this literature, so as to ascertain that these claims are true to these two models' mode of operation and capabilities. Special attention should be paid to the obvious, and not so obvious, incongruities between the rhetoric on the severity of the uncertainty that info-gap decision theory is claimed to address and the actual capabilities of this method to handle such an uncertainty.

All in all, as far as the peer review is concerned, stricter requirements should be applied to articles on info-gap decision theory to ensure that claims made about this theory be substantiated by solid proofs and by citation of relevant references. Referees of articles on info-gap decision theory should be familiar with the technical aspects of worst-case analysis, maximin analysis and robust optimization.

The list of hard facts about info-gap decision theory in Section 12 speaks for itself. Referees of articles on info-gap decision theory should consult this list.

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